From Chapter 1

A function f is called **increasing** on an interval I if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

It is called **decreasing** on I if

$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

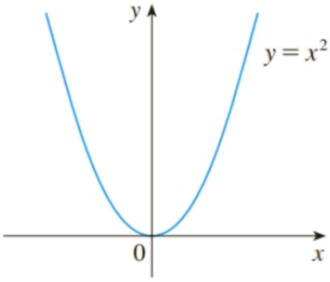


FIGURE 23

You can see from Figure 23 that the function $f(x) = x^2$ is decreasing on the interval $(-\infty, 0]$ and increasing on the interval $[0, \infty)$.

Increasing/Decreasing Test

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

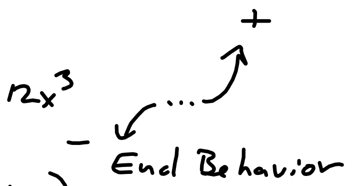
EXAMPLE 1 Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and where it is decreasing.

$\Longrightarrow f'(x) = 12x^3 - 12x^2 - 24x \overset{\text{SET}}{=} 0$

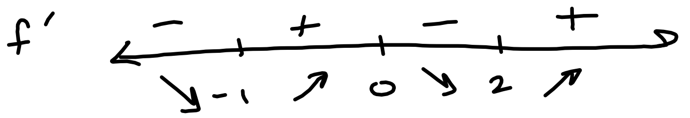
$\Longrightarrow 12x(x^2 - x - 2) = 0$

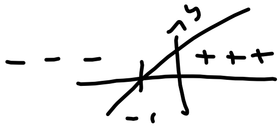
$\Longrightarrow 12x(x-2)(x+1) = 0$

$\Longrightarrow x = -1, 0, 2$ (CRITICAL #s)



My Way:

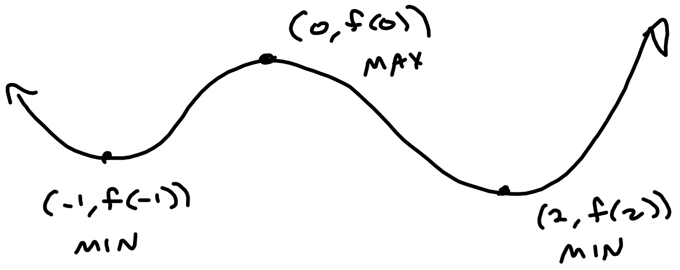




$12x(x+1)(x-2)$

manage sign changes

$\Longrightarrow$

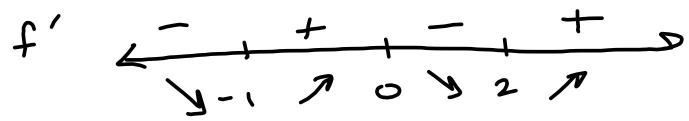


We know where the local max/min points are. We haven't found their y-values, yet.

Book Way is painful.

	$x+1$	x	$x-2$	$f'(x)$
$x < -1$	−	−	−	−
$-1 < x < 0$	+	−	−	+
$0 < x < 2$	+	+	−	−
$2 < x$	+	+	+	+

You get the same sign pattern for f'.



Preview of 1st Derivative Test for Max/Min points on a graph.

Where the up arrow meets a down arrow is a max.

Where the down arrow meets an up arrow is a min!

The book says that the 1st derivative in the example shows the function is decreasing on $(-\infty,-1) \cup (0,2)$ &

increasing on $(-1,0) \cup (2,\infty)$

Technically, decreasing on $(-\infty,-1] \cup [0,2]$

& increasing on $[-1,0] \cup [2,\infty)$

Critical Number for f.

$x \in D(f)$ ($f(x)$ defined)

Either $f'(x) = 0$ or $f'(x)$ undefined.

→ f is going vertical $\frac{\text{stuff}}{0} = f'(x)$

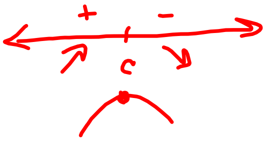
The First Derivative Test Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' is positive to the left and right of c, or negative to the left and right of c, then f has no local maximum or minimum at c.

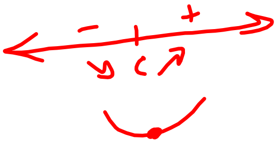
ⓐ f'



$x=c$ is critical #

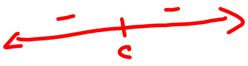
MAX

ⓑ f'



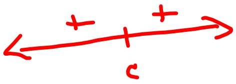
MIN

ⓒ f'



NEITHER

ⓒ



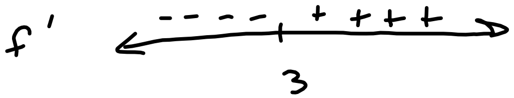
Critical #s for $f(x)$ in example 1 were

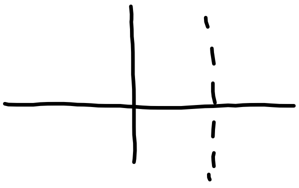
$x = -1, 0, 2$

Critical #s where f'(c) doesn't exist

$f(x) = (x-3)^{\frac{2}{3}}$

$f'(x) = \frac{2}{3}(x-3)^{-\frac{1}{3}}(1) = \frac{2}{3(x-3)^{1/3}}$ x=3 is critical

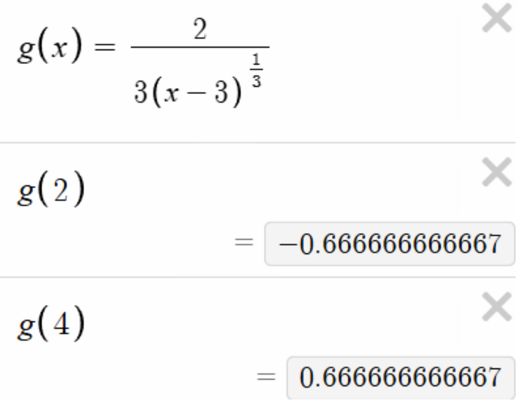




Not sure? Use test values.

	Test	
$x<3$	2	$\frac{2}{3(2-3)^{\frac{1}{3}}}$
$x>3$	4	

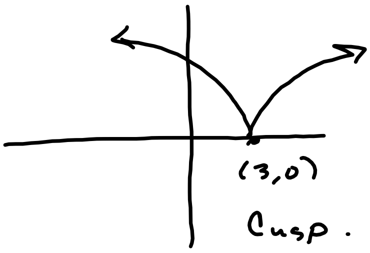
$\frac{2}{3(x-3)^{\frac{1}{3}}}$



$g(x) = \frac{2}{3(x-3)^{\frac{1}{3}}}$

$g(2)$ = −0.666666666667 ↘

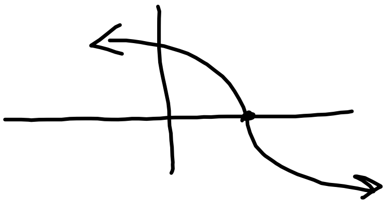
$g(4)$ = 0.666666666667 ↗



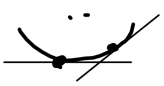
$\boxed{f(x) = -(x-3)^{\frac{3}{4}}}$ → Not quite

Looking for one of these;

Can't think of one off the top.



Definition If the graph of f lies above all of its tangents on an interval I, then it is called **concave upward** on I. If the graph of f lies below all of its tangents on I, it is called **concave downward** on I.


$f'' > 0$

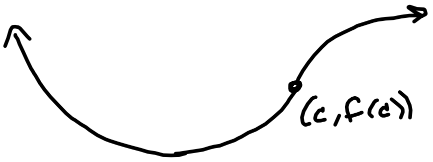

$f'' < 0$

Concavity Test

(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

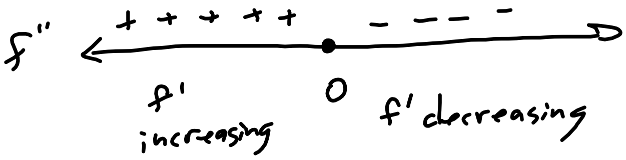
(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Definition A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.



Inflection Point!

$f'' = 0$, f'' undefined.



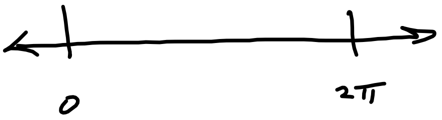
$f(x) = \sin(x)$

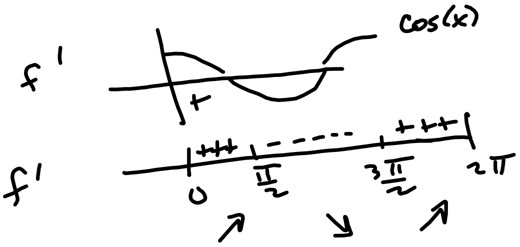
Graph on $[0, 2\pi]$

$f'(x) = \cos(x)$

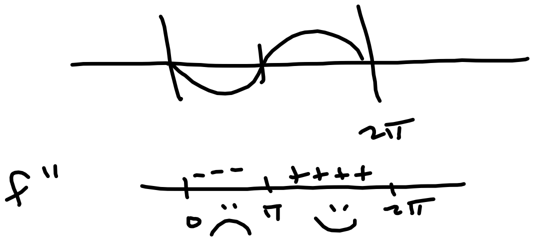
$f''(x) = -\sin(x)$

$f'(x) = \cos(x)$

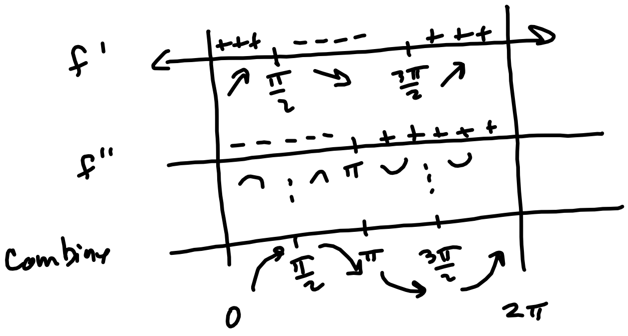


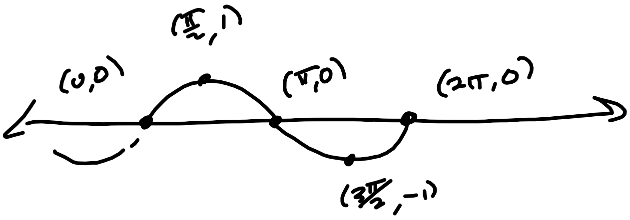


$f''(x) = -\sin(x)$



Combine:





$(0,0)$ IP

$(\frac{\pi}{2},1)$ MAX

$(\pi,0)$ IP

$(\frac{3\pi}{2},-1)$ MIN

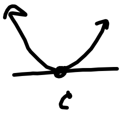
$(2\pi,0)$ IP

Complete graph shows all max/min points, inflection points and vertical/horizontal/oblique asymptotes.

The Second Derivative Test for max/min.

$f'(c)=0$ and $f''(c)>0$

$f'(c)=0$



MIN

$f'(c)=0$ and $f''(c)<0$

MAX

The Second-Derivative Test is a good check, but 1st-Derivative Test is pretty much all you need to determine max/min.