Quiz 2 Chapter 2 Part II #1

$$\boxed{\frac{3y - x}{y - 3x}}$$

Find dy/dx by implicit differentiation.

$$x^2 - 6xy + y^2 = 6$$

$$2x - 6y - 6xy' + 2yy' = 0$$

$$(-6x + 2y)y' = 6y - 2x$$

$$y' = \frac{6y - 2x}{2y - 6x} = \boxed{\frac{3y - x}{y - 3x} = \frac{dy}{dx}}$$

Midterm Bonus Section

2. (5 pts) Compute the derivative of $f(x)=x^{\frac{4}{3}}$ by the limit definition of the derivative. Hint: Use this formulation: $\lim\limits_{x\to c}\dfrac{f(x)-f(c)}{x-c}$ and be mindful of special products.

$$\frac{f(x)-f(c)}{x-c} = \left(\frac{x^{\frac{4}{3}}-c^{\frac{4}{3}}}{x-c}\right)\left(\frac{x^{\frac{8}{3}}+x^{4/3}c^{4/3}+c^{8/3}}{x^{8/3}+(xc)^{4/3}+c^{8/3}}\right) = \frac{(a-b)(a^2+ab+b^2)}{\sim\sim}$$

$$= \frac{a^3-b^3}{\sim\sim} = \frac{x^4-c^4}{(x-c)(x^{8/3}+(xc)^{4/3}+c^{8/3})} = \frac{(x^2-c^2)(x^2+c^2)}{\sim\sim}$$

$$= \frac{\cancel{(x-c)}(x+c)(x^2+c^2)}{\cancel{(x-c)}(x^{8/3}+(xc)^{4/3}+c^{8/3})} \xrightarrow{x\to c} \frac{(c+c)(c^2+c^2)}{c^{8/3}+c^{8/3}+c^{8/3}}$$

$$= \frac{(2c)(2c^2)}{3c^{\frac{8}{3}}} = \frac{4c^3}{3c^{\frac{8}{3}}} = \frac{4}{3}c^{3-\frac{8}{3}} = \boxed{\frac{4}{3}c^{\frac{1}{3}}}$$

$$\frac{d}{dx}\left[x^{\frac{4}{3}}\right] = \frac{4}{3}x^{\frac{4}{3}-1} = \frac{4}{3}x^{\frac{1}{3}}$$

1. (5 pts) Prove that $\lim_{x\to 2}(3x^2-2x-5)=3$, using the $\varepsilon-\delta$ definition of limit.

Let $\varepsilon>0$ be given. We need $\delta>0 \ni |3x^2-2x-5-3|<\varepsilon$

whenever $0<|x-2|<\delta$.

Assume $\delta\le 1$. Then

$|3x^2-2x-5-3| = |3x^2-2x-8| = |x-2||3x+4| < |3x+4|\delta$

Now $|3x+4|$ needs a bound

$1 < x < 3$

$3 < 3x < 9$

$7 < 3x+4 < 13$

$\Longrightarrow |3x+4| < 13.$

Now, Define $\delta = \min\{1, \frac{\varepsilon}{13}\}$. Then $0<|x-2|<\delta$

$\Longrightarrow |3x^2-2x-5-3| = |3x^2-2x-8| = |3x+4||x-2| < 13\delta \le \varepsilon$ ■

<u>Claim:</u> $\lim_{x \to 3} (2x+7) = 13$

<u>Proof:</u> want $\delta \ni 0 < |x-c| < \delta \Longrightarrow |2x+7-13| < \varepsilon$

iff $\Longleftrightarrow |2x-6| < \varepsilon$

$\Longleftrightarrow 2|x-3| < \varepsilon$

$\Longleftrightarrow |x-3| < \frac{\varepsilon}{2}$

Define $\delta = \frac{\varepsilon}{2}$ ■