Extreme Value Theorem

Fermat's Theorem

Rolle's Theorem

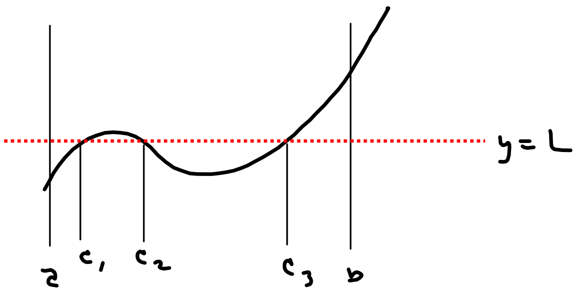
Mean Value Theorem

f is cnt's @ x = a means $\lim_{x \to} f(x) = f(a)$

Recall: Intermediate Value Theorem

f cnt's on [a, b] $\Longrightarrow \exists\, c \in (a,b) \ni f(c) = L$

for any L that's between f(a) & f(b).



$f(a) < L < f(b)$

$f(a) > L > f(b)$

This doesn't tell you where *c* is. It just tells you there *is* a *c* that satisfies the conclusion.

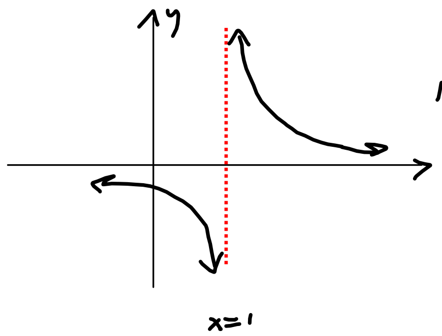
Extreme Value Theorem

If f is cnt^s on [a,b], then f achieves a maximum and a minimum on [a,b]

Nonexamples.

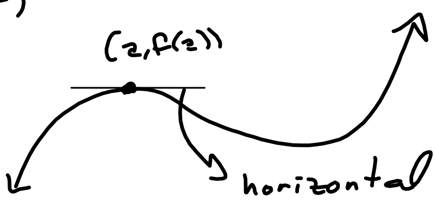
$f(x) = \frac{1}{x-1}$ on [0,2] blows up @ $x=1$.

There's no bound.



No min/max on [0,2]

Fermat's Theorem If $(a, f(a))$ is a LOCAL max/min and f is differentiable (difb^l), then $f'(a) = 0$.



horizontal tangent at $(a, f(a))$
$L_a(x) = f(a)$

This is huge for finding where a differentiable function achieves a max/min. We're using the converse of the theorem to do so.

f difbl & $f(a)$ is extreme $\Rightarrow f'(a) = 0$

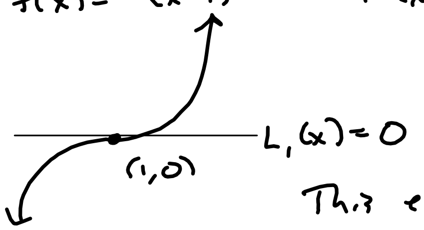
Converse: NOT logically equivalent

$f'(a) = 0 \Rightarrow f(a)$ is extreme.

This doesn't hold in all cases but it gives us candidates for extremes.

Non example

$f(x) = (x-1)^3$



$f'(x) = 3(x-1)^2(1) = 3(x-1)^2 \overset{SET}{=} 0$

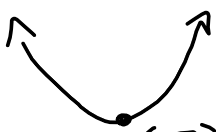
$\Longrightarrow x=1$ is a candidate, but doesn't win the election.

This exception to the converse is a "terrace point."

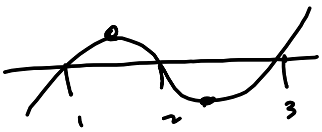
In general practice:

Find the vertex of $f(x) = (x-1)^2 + 7$

$f'(x) = 2(x-1)^1(1) = 2(x-1) \overset{SET}{=} 0 \Longrightarrow x=1$



(1,7) is local (and Absolute) MIN.



cubic

$(x-1)(x-2)(x-3)$

use quadratic formula.

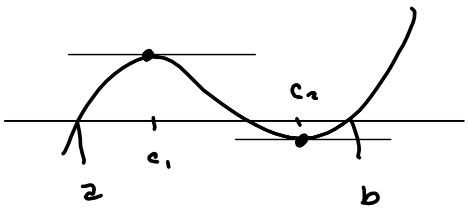
Converse of "A implies B" is "B implies A."

These two are not equivalent. I can't make it rain by using an umbrella.

Contrapositive of "A implies B" is "NOT-B implies NOT-A."

If I'm not using an umbrella, then it's not raining.

Rolle's Theorem f cnt^s on $[a,b]$ and f is difb^l on (a,b) and $f(a)=f(b) \Longrightarrow \exists\, c \in (a,b) \ni f'(c)=0.$



for each, for all, for every

Rough Proof

$f(x)=f(a)$ everywhere. Then $f'(c)=0 \;\forall\; c \in (a,b)$.

Suppose $f(x)$ is not constant.

There's either a point where $\underset{(i)}{f(x) > f(a)}$ or $\underset{(ii)}{f(x) < f(a)}$

By EVT, f achieves a maximum on $[a,b]$

Clearly $f(a)$ isn't max. $f(b)=f(a)$ isn't the max,

so $\exists\, c \in (a,b) \ni f(c)$ is a max.

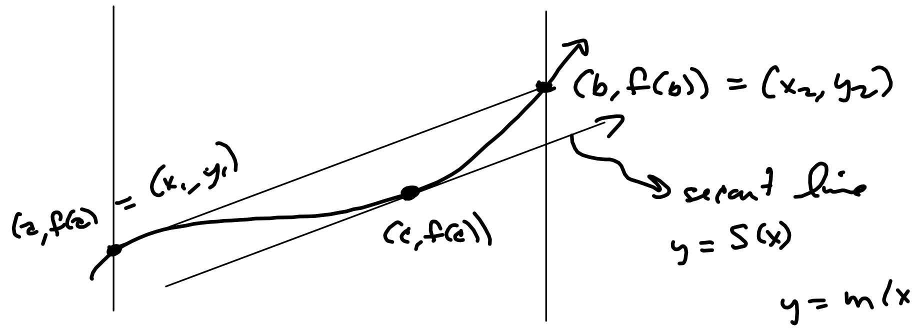
$\Longrightarrow f'(c) = 0$ ■ (Case (ii): Replace "max" with "min."

Mean Value Theorem

f cnt^s $[a,b]$, f difb$^{\underline{l}}$ on (a,b) $\longrightarrow$

$\exists\, c \in (a,b) \ni f'(c) =$ Average slope $= m_{sec} = \dfrac{f(b)-f(a)}{b-a}$

$= \dfrac{y_2-y_1}{x_2-x_1}$



$y = m(x-x_1)+y_1$

$= m_s(x-a)+f(a)$

$= \dfrac{f(b)-f(a)}{b-a}(x-a)+f(a)$

Note: $f(x) = S(x)$ @ $x=a$ & $x=b$

Define $h(x) = f(x) - S(x)$

$$= f(x) - \frac{f(b)-f(a)}{b-a}(x-a) + f(a)$$

$h(x)$ is cnt^{s} & difb^{l} b/c f & S

$h(a) = 0$, $h(b) = 0$

$$h(a) = f(a) - \left(\left(\frac{f(b)-f(a)}{b-a}\right)(a-a) + f(a)\right) = 0$$

$$h(b) = f(b) - \left(\left(\frac{f(b)-f(a)}{b-a}\right)(b-a) + f(a)\right)$$

$$= f(b) - \left(f(b) - f(a) + f(a)\right) = 0 \longrightarrow$$

Rolle's twoen applies to $h(x)$. $\longrightarrow \exists\, c \in (a,b) \ni$

$h'(c) = 0$

$$h'(x) = f'(x) - \left(\frac{f(b)-f(a)}{b-a}\right) \quad \&$$

$$h'(c) = f'(c) - \frac{f(b)-f(a)}{b-a} = 0 \longrightarrow$$

$$f'(c) = \frac{f(b)-f(a)}{b-a} \quad \blacksquare$$

As with Rolle's, there's no single formula to find *c*. You just know there*is* a *c*.

This is the end of new stuff for the Midterm.

6. (5 pts) Let $f(x)=x^3-6x^2+5x-4$ Find $c\in(0,4)$ such that $f'(c)$ equals the average slope of f over the interval $[0,4]$. How did you know such a c existed before you started?

f is cnt³ and difbl Everywhere, so hypotheses of MVT hold.

$f(0)=-4$

$f(4)=-16$

$m_{sec}=\frac{f(b)-f(a)}{b-a}=\frac{-16-(-4)}{4-0}$

$=\frac{-16+4}{4}=\frac{-12}{4}=-3=m_{sec}.$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 5 & -4 \\ & & 4 & -8 & -12 \\ \hline & 1 & -2 & -3 & -16 \end{array}$$

$f'(x)=3x^2-12x+5 \overset{SET}{=} -3$

$\Longrightarrow 3x^2-12x+8=0$

$3(x^2-4x+2^2) = -8+12$

$3(x-2)^2=4$

$(x-2)^2=\frac{4}{3} \longrightarrow$

$x=2\pm\sqrt{\frac{4}{3}}=2\pm\frac{2}{\sqrt{3}}$ ≈

Let $c=2+\frac{2}{\sqrt{3}}$ OR $2-\frac{2}{\sqrt{3}}$

$a=3,\ b=-12,\ c=8$

$b^2-4ac=12^2-4(3)(8)$

$=144-96=48$

$x=\frac{12\pm\sqrt{48}}{2(3)}=\frac{12\pm4\sqrt{3}}{6}$

$=\frac{6\pm2\sqrt{3}}{3}$

$=2\pm\frac{2\sqrt{3}}{3}$ ✓