6. (5 pts) Prove that the equation $f(x)=x^4-4x^3+6x^2+28x-91$ has a root in the interval $(0,5)$, but *do not solve*!

function

Prove that the equation $x^4-4x^3+6x^2+28x-91=0$ has a solution in the interval $(0,5)$

$f(x)$ is a polynomial $\implies$ cnt^s everywhere.

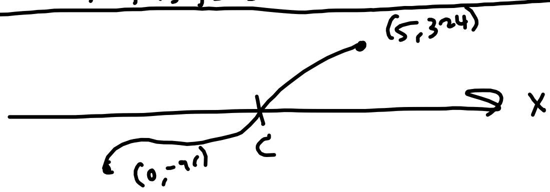
$f(0)=91$ → -91, dingbat! and $f(5)=324$

Doh!

5	1	−4	+6	28	−91
		5	5	55	415
	1	1	11	83	324

By IVT

$$\boxed{0>f(0)=-91 \ \& \ f(5)=324>0 \implies \exists\, c\in(0,5) \ni f(c)=0}$$



7. Differentiate the following with respect to the indicated independent variable. **Do not simplify!**

a. (5 pts) $f(x) = \sqrt[3]{x^5} - 27x^{\frac{7}{4}} + -\dfrac{11}{x^6};\ x.$

$= x^{5/3} - 27x^{\frac{7}{4}} - 11x^{-6} \longrightarrow$

$\boxed{f'(x) = \frac{5}{3}x^{2/3} - \frac{189}{4}x^{\frac{3}{4}} + 66x^{-7}}$

$\frac{5}{3} - \frac{3}{3} = \frac{2}{3}$

$\left(\frac{7}{4}\right)(27) = \frac{189}{4}$

b. (5 pts) $g(x) = \cos(5x)\sec(3x);\ x.$

$g(x) = \cos(5x)\sec(3x) \longrightarrow$

$g'(x) = -\sin(5x)\cdot 5\cdot \sec(3x) + \cos(5x)\sec(3x)\tan(3x)\cdot 3$ FINE

$\frac{dy}{du}\cdot\frac{du}{dx} = \frac{d(\sec(3x))}{d(3x)}\cdot\frac{d(3x)}{dx}$

$= -5\sin(5x)\sec(3x) + 3\cos(5x)\sec(3x)\tan(3x)$ MORE EXPERIENCED

c. (5 pts) $h(\beta) = \dfrac{12\beta^3 + 2\beta}{\tan(\beta)}$; β.

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} = \frac{(36\beta^2+2)(\tan(\beta)) - (12\beta^3+2\beta)(\sec^2(\beta))}{\tan^2(\beta)}$$

Curveball:

$$h(\beta) = \frac{12\beta^2 + 2\beta}{\tan(\beta)} \;;\; x$$

$$\Rightarrow \frac{dh}{dx} = 0 \,!$$

8. Consider the relation $x^2 - 3xy + 4y^2 = \cos(y)$.

a. (5 pts) Use implicit differentiation to find $y' = \frac{dy}{dx}$

b. (5 pts) Find an equation of the tangent line to the curve at the point $(1,0)$.

(a) $2x - 3y - 3xy' + 8yy' = -\sin(y)\,y'$ Sam says $+3y$

$\Rightarrow [-3x + 8y + \sin(y)]\,y' = -2x - 3y \Rightarrow$

$y' = \dfrac{-2x - 3y}{-3x + 8y + \sin(y)}$ $\to +3y$!

He is correct!

(b) $(x_1, y_1) = (1, 0) \Rightarrow$

$y' = \dfrac{-2}{-3+0+0} = \dfrac{2}{3} \Rightarrow$

$\boxed{L(x) = y = \frac{2}{3}(x-1) + 0}$

$\left(y = m(x - x_1) + y_1 \right.$
$\left. = y'(x_1)(x - x_1) + y_1 \right)$

10. (10 pts) If the minute hand of a clock has length r (in centimeters), find the rate at which it sweeps out area as a function of r.

Don't forget lexicon:

$A =$ area swept by minute-hand (in cm²) as a function of $t =$ time.

3 out of 10 for Lex.

$A = \frac{1}{2}r^2\theta$

Want $\frac{dA}{dt}$ as function of r.

$$\frac{dA}{dt} = r\frac{dr}{dt}\theta + \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt}$$

Chain rule.

Note: r is fixed! $\Longrightarrow \frac{dr}{dt} = 0$

Minutes:

$$\left(\frac{\text{One rev}}{60 \text{ min}}\right)\left(\frac{2\pi \text{ radians}}{\text{One rev}}\right) = \frac{\pi}{30} \text{ radians/min}$$

$$\Longrightarrow \frac{dA}{dt} = \frac{1}{2}r^2 \cdot \frac{\pi}{30} = \boxed{\frac{\pi r^2}{60} \frac{\text{cm}^2}{\text{min}}}$$

$$\boxed{\text{or } \frac{\pi r^2 \text{ cm}^2}{\text{hr}}}$$

1. (10 pts) Evaluate $\lim\limits_{x\to 3} \dfrac{x^2-4}{x^3-8}$ by factoring. $x\to 2$

$\lim\limits_{x\to 2} \dfrac{x^2-4}{x^3-8} =$

$= \lim\limits_{x\to 2} \dfrac{(x-2)(x+2)}{(x-2)(x^2+2x+4)} = \lim\limits_{x\to 2} \left(\dfrac{x+2}{x^2+2x+4}\right) = \dfrac{2+2}{2^2+2(2)+4} = \dfrac{4}{12} \boxed{= \dfrac{1}{3}}$

2. (10 pts) Evaluate each of the following by factoring and simplifying. One exists. The other doesn't.

a. $\lim\limits_{x\to -7} \dfrac{3x^2+17x-28}{4x^2+31x+21}$

$4(-7)^2+31(-7)+21 = 196-217+21 = 0$ ✓

$3(-7)^2+17(-7)-28$

$= \cancel{147} - 119 - 28$

$147-147=0$ ✓

$\begin{array}{r} 49 \\ \times 4 \\ \hline 196 \end{array}$ $\begin{array}{r} 31 \\ \times 7 \\ \hline 217 \end{array}$ $\begin{array}{r} 49 \\ \times 3 \\ \hline 147 \end{array}$

-7 is a zero of numerator & denominator!

$\begin{array}{r|rrr} -7 & 3 & 17 & -28 \\ & & -21 & 28 \\ \hline & 3 & -4 & 0 \end{array}$

$\begin{array}{r|rrr} -7 & 4 & 31 & 21 \\ & & -28 & -21 \\ \hline & 4 & 3 & 0 \end{array}$

$\dfrac{3x-4}{4x+3} \xrightarrow{x\to -7} \dfrac{3(-7)-4}{4(-7)+3} = \dfrac{-25}{-25} = \boxed{1}$

My usual, blunt-force Method:

Sledgehamer:

$3x^2+17x-28 \overset{\text{SET}}{=} 0$

$a=3,\ b=17,\ c=-28$

$b^2-4ac = 17^2-4(3)(-28) = 289+336 = 625 = 25^2$!

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = \dfrac{-17\pm\sqrt{625}}{2(3)} = \dfrac{-17\pm 25}{6} = \dfrac{8}{6} = \dfrac{4}{3}$, $\dfrac{-42}{6} = -7$

$3(x+7)(x-\tfrac{4}{3})$

$\begin{array}{r} 17 \\ \times 17 \\ \hline 119 \\ 170 \\ \hline \end{array}$ $\begin{array}{r} 280 \\ 56 \\ \hline 336 \end{array}$

$4x^2+31x+21=0$

$a=4,\ b=31,\ c=21$

$b^2-4ac = 31^2-4(4)(21) = 961-336 = 625 = 25^2$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = \dfrac{-31\pm 25}{2(4)} = \dfrac{-31\pm 25}{8}$: $-\dfrac{6}{8} = -\dfrac{3}{4}$, $\dfrac{-56}{8} = -7$

$\boxed{4(x+\tfrac{3}{4})(x+7)}$

$\begin{array}{r} 16 \\ \times 21 \\ \hline 16 \\ 320 \\ \hline 336 \end{array}$ $\begin{array}{r} 31 \\ \times 31 \\ \hline 31 \\ 93 \\ \hline 961 \end{array}$ $7\overline{)961}$ 13

$\dfrac{3\cancel{(x+7)}(x-\frac{4}{3})}{4\cancel{(x+7)}(x+\frac{3}{4})} \xrightarrow{x\to -7} \dfrac{3(-7-\frac{4}{3})}{4(-7+\frac{3}{4})} = \dfrac{-21-4}{-28+3} = \dfrac{-25}{-25} = 1$