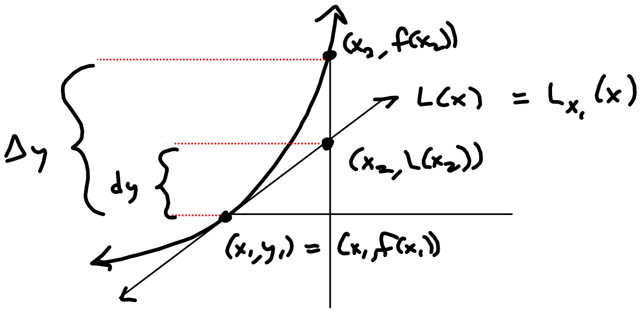
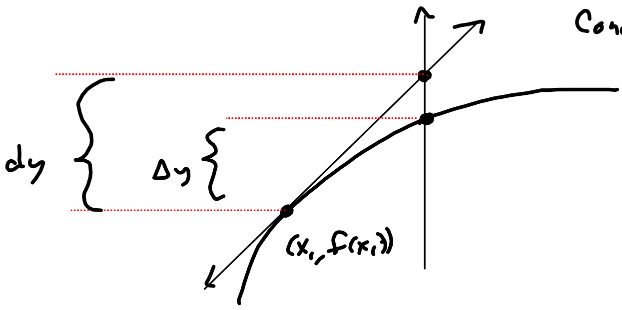
Hit "Record."

Concave Up : $\Delta y > dy$
to the right



Concave Down : $dy > \Delta y$
(to the right)



Tangent Line

$$f(x) \approx L(x) = \underbrace{f'(x_1)(x-x_1)}_{\text{Change in } L(x)} + \underbrace{f(x_1)}_{\text{Starting height}}$$

$$\text{Change in } L(x) = f'(x_1)(x-x_1) = dy$$

$$dy = \text{Differential of } y = f'(x)\,dx$$

$$\Delta y = f(x) - f(x_1) \approx f'(x)\,dx = f'(x)\,\Delta x$$

dx & Δx = small change in the input (argument).

2.9 #13

The radius of a circular disk is given as 23 cm with a maximum error in measurement of 0.2 cm.

(a) Use differentials to estimate the maximum error in the calculated area of the disk. (Round your answer to two decimal places.)

✗ 28.90 cm^2

(b) What is the relative error? (Round your answer to four decimal places.)

✗ 0.0174

What is the percentage error? (Round your answer to two decimal places.)

✗ 1.74 %

r = radius of disk, in cm

A = area of the disk, in cm^2

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$dA = 2\pi r\, dr$$

$$r = 23 \text{ cm} \pm 0.2 \text{ cm}$$

$$dr = \Delta r = \pm .2 \text{cm}$$

Error Estimate, based on error in measurement of radius

$$\Delta A \approx dA = 2\pi(23)(\pm 0.2) = \pm 9.2\pi \text{ cm}^2 \approx 28.902652413$$

$$\approx \boxed{28.90 \text{ cm}^2 \approx \Delta A}$$

Relative Error:

$$\frac{\Delta A}{A} \approx \frac{dA}{A} \approx \frac{28.90}{\pi(23)^2} \approx 0.0173897083378 \approx \boxed{0.0174}$$

$$\text{Percent Error} = \left(\frac{\Delta A}{A}\right)(100\%) \approx \boxed{1.74\%}$$

Find $\sqrt{97}$ using tangent line approximation.

$x_1 = 100$

$x_2 - x_1 = 97 - 100 = -3 = \Delta x \equiv dx$

$L(x) = f'(x_1)(x - x_1) + f(x_1)$

$f(x) = \sqrt{x} = x^{\frac{1}{2}} \longrightarrow$

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f'(x_1) = \frac{1}{2\sqrt{100}} = \frac{1}{2(10)} = \frac{1}{20}$

$f(x_1) = f(100) = 10$

$\longrightarrow L(x) = f(x_1)(x - x_1) + f(x_1)$

$= \frac{1}{20}(x - 100) + 10$

$= \frac{1}{20}(97 - 100) + 10$

$= \frac{1}{20}(-3) + 10$

$= -\frac{3}{20} + 100 = \frac{-3 + 2000}{20} = \frac{1997}{20}$

$\frac{1}{20} \cdot (-3) + 10$

$= 9.85$

$\sqrt{97}$

$= 9.8488578018$

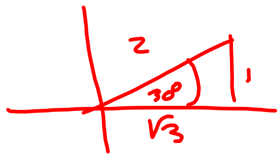
For trig functions, this stuff only works if you're in radians.

Give a tangent-line approximation for $\sin(32^\circ)$

$f(x) = \sin(x)$

$f'(x) = \cos(x)$

$x_1 = 30^\circ = (30^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{6} = x_1$



$x_2 = 32^\circ = (32^\circ)\left(\frac{\pi}{180^\circ}\right)$ Convert

$L(x) = f'(x_1)(x - x_1) + f(x_1)$ $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$= \sin\left(\frac{\pi}{6}\right)\left(\frac{32\pi}{180} - \frac{30\pi}{180}\right) + \sin\left(\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2}\left(\frac{32\pi - 30\pi}{180}\right) + \frac{1}{2}$

$= \frac{\sqrt{3}}{2}\left(\frac{2\pi}{180}\right) + \frac{1}{2} = \frac{\sqrt{3}\pi}{180} + \frac{1}{2}$

$$\frac{\sqrt{3}}{2}\left(\frac{32\pi}{180} - \frac{30\pi}{180}\right) + \frac{1}{2}$$

$= 0.530229989404$ Tangent Line Approx

$\sin(32)$

$= 0.529919264233$ "Actual"