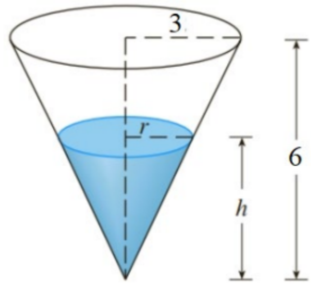
Related Rates from the Midterm last spring.

9. (10 pts) A water tank has the shape of an inverted circular cone with base radius 3 m and height 4 m. If water is being pumped out of the tank at a rate of 3 $\frac{\text{m}^3}{\text{min}}$, find the rate at which the water level is dropping when the water is 4 m deep. Hints: The volume of a circular cone is $V = \frac{1}{3}\pi r^2 h$. It is possible (and essential) to write V as a function of r for this situation.



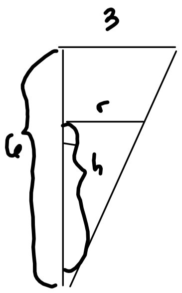
r = radius of the top of the water

h = height/depth of the water (in m)

V = volume of water in the cone (m^3)

Given: $\frac{dV}{dt} = -3 \frac{m^3}{min}$

Want $\left.\frac{dh}{dt}\right|_{h=4}$.



$\frac{r}{h} = \frac{3}{6} \Longrightarrow r = \frac{1}{2}h$ by similar triangles.

(in m)

~~$\Longrightarrow h = 2r$ is better.~~

Now $V = \frac{1}{3}\pi r^2 h \Longrightarrow$

$= \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$= \frac{1}{3}\pi\left(\frac{h^2}{4}\right)h = \frac{1}{12}\pi h^3 \Longrightarrow$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \cdot \frac{dh}{dt} \overset{\text{SET}}{=} -3 \Longrightarrow$

$\frac{dh}{dt} = -3(4)\left(\frac{1}{\pi}\right)\left(\frac{1}{h^2}\right)$

Need $\Longrightarrow \left.\frac{dh}{dt}\right|_{h=4} = \frac{-12}{\pi} \cdot \frac{1}{4^2} = -\frac{12}{16\pi} = \boxed{-\frac{3}{4\pi} \frac{m}{min}}$

$\boxed{= \left.\frac{dh}{dt}\right|_{h=4}}$

3. (5 pts) Simplify the limit of the difference quotient $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ for $f(x)=x^2+2x-10$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+2(x+h)-10-(x^2+2x-10)}{h}$$

$$= \frac{x^2+2xh+h^2+2x+2h-10-x^2-2x+10}{h} = \frac{2xh+h^2+2h}{h} = \frac{h(2x+h+2)}{h}$$

$$= 2x+h+2 \xrightarrow{h\to 0} \boxed{2x+2 = f'(x)}$$

$(h \neq 0)$

4. The point $P(1,-7)$ lies on the graph of $f(x)=x^2+2x-10$.

a. (5 pts) Write the equation of the tangent line $L_1(x)$ to $f(x)$ at $x=1$.

b. (5 pts) Sketch a graph of $f(x)$ and $L_1(x)$ on the same set of coordinate axes.

a) $(a, f(a)) = (1,-7)$

$f'(a) = f'(1) = 2(1)+2 = 4$

$L(x) = f'(a)(x-a)+f(a)$

$= f'(a)(x-1)+(-7)$

$= \boxed{4(x-1)-7 = L(x)}$

b) Graph of $x^2+2x-10$ & tan line @ $(1,-7)$:

Vertex:

M1:

$f'(x) = 2x + 2 \overset{\text{SET}}{=} 0$

$2x = -2$

$x = -1$

$f(-1) = (-1)^2 + 2(-1) - 10$

$= 1 - 2 - 10$

$= -11 \rightarrow (h, k) = (-1, -11)$

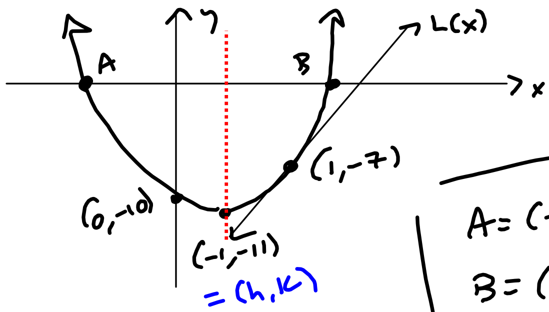
M2

$x^2 + 2x - 10$

$= x^2 + 2x + 1^2 - 1^2 - 10$

$= (x+1)^2 - 11 \rightarrow$

$(h, k) = (-1, -11)$



$$\boxed{\begin{aligned} A &= (-1-\sqrt{11}, 0) \\ B &= (-1+\sqrt{11}, 0) \end{aligned}}$$

A & B: $x^2 + 2x - 10 = 0$

$x^2 + 2x = 10$

$x^2 + 2x + 1^2 = 10 + 1^2$

$(x+1)^2 = 11$

$x + 1 = \pm\sqrt{11}$

$x = -1 \pm \sqrt{11}$ can also be used to find vertex:

M3 $x = -1 \pm \sqrt{11}$ gives x-ints

use symmetry: $\dfrac{-1+\sqrt{11} + -1-\sqrt{11}}{2} = \dfrac{-2}{2} = -1 = h$

Vertex is in the middle (on the axis of symmetry)