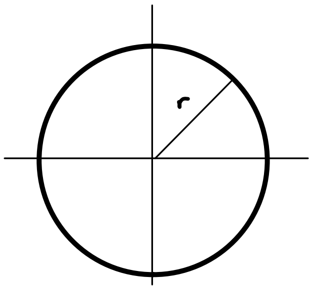
Section 2.8 - Related Rates. CHAIN RULE. ALSO, IMPLICIT DIFFERENTIATION.

Area is a function of *r* and *r* is a function of *t*.

Find the rate at which the area of an oil spill is growing, if the radius is growing at a rate of 2 m/s.



A = Area in square meters
r = radius of oil spill (in meters)
t = time, in seconds.

(a) If A is the area of a circle with radius r and the circle expands as time passes, find dA/dt in terms of dr/dt.

$A = \pi r^2 = \pi (r(t)^2)$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$, by Chain Rule.

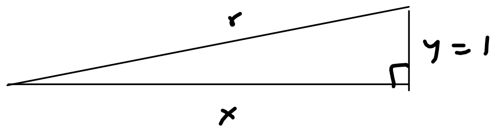
$A' = 2\pi r r'$

(b) Suppose the radius is growing at 2 m/s. Find dA/dt when radius = 24 m.

Find $\left.\frac{dA}{dt}\right|_{r=24} = \left. 2\pi r \cdot r' \right|_{r=24} = 2\pi(24)(2) = 96$ m^2/s

2.8 #10

A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 7 m from the dock? (Round your answer to two decimal places.)



Let x = distance to dock (m)

Want $\left.\frac{dx}{dt}\right|_{x=7}$, given $\frac{dr}{dt} = 1$

$$x^2 + 1^2 = r^2$$

$$2x\frac{dx}{dt} = 2r\frac{dr}{dt}$$

$$2(7)\frac{dx}{dt} = 2r \cdot 1$$

We need $\left. r\right|_{x=7}$

$$x^2 + 1^2 = r^2$$

$$7^2 + 1^2 = 49 + 1 = 50 = r^2 \Longrightarrow$$

$$r = \sqrt{50} = 5\sqrt{2} \longrightarrow$$

$$14\frac{dx}{dt} = 2(5\sqrt{2}) = 10\sqrt{2} \longrightarrow$$

$$\frac{dx}{dt} = \frac{10\sqrt{2}}{14} = \frac{5\sqrt{2}}{7} \approx 1.01015254455 \approx \boxed{1.01 \text{ m/s} \approx \frac{dr}{dt}}$$

1.01 m/s