$$\frac{d}{dx}\left[x^5\right] = 5x^4$$

$$\frac{d}{d\theta}\left[\theta^5\right] = 5\theta^4$$

$$\frac{d}{dz}\left[x^5\right] = 0$$

$\sin^{-1}(\theta) = \frac{1}{\sin\theta}$ No!

$\sin^{-1}(\theta) = \arcsin\theta$

$$\frac{d}{d\sin\theta}\left[(\sin\theta)^5\right] = \frac{d}{d\sin\theta}\left[\sin^5\theta\right] = 5\sin^4\theta$$

Chain Rule:

$$\frac{d}{dx}\left[\sin^5(x)\right] = \left(5\sin^4(x)\right)\left(\frac{d}{dx}\left[\sin(x)\right]\right)$$

$$= 5\sin^4(x)\cdot\cos(x)$$

$$\frac{d}{dx}\left[(\sin(x))^5\right] = 5\sin^4(x)\cdot\cos(x)$$

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{d}{d\,g(x)}\left[f(g(x))\right]\cdot\frac{d}{dx}\left[g(x)\right]$$

$$\frac{d}{dx}\left[(x^3-5x)^2\right] = \frac{d}{d(x^3-5x)}\left[(x^3-5x)^2\right]\cdot\frac{d}{dx}\left[x^3-5x\right]$$

$$= 2(x^3-5x)^2\cdot(3x^2-5)$$

$$\frac{d}{dx}\left[f(u(x))\right] = \frac{df}{du}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left[(x^3-5x^2)^2\right] = \frac{d}{dx}\left[x^6-10x^5+25x^4\right] = 6x^5-50x^4+100x^3$$

$$2(x^3-5x)^2\cdot(3x^2-5) = 2\left[3x^5-15x^3-5x^3+25x\right]$$

$$= 6x^5-40x^3+50x$$

$$(x^3-5x)^2 = x^6 - 2(x^3)(5x) + 25x^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Why chain Rule?

$$\frac{d}{dx}\left[(x^3-5x)^{17}\right] = 17(x^3-5x)^{16}(3x^2-5)$$

$$\frac{d}{dx}\left[\sin(5x)\right] = \frac{d\ \sin(5x)}{d\ 5x} \cdot \frac{d\ 5x}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

$f(u) = \sin(u)$

$u(x) = 5x$

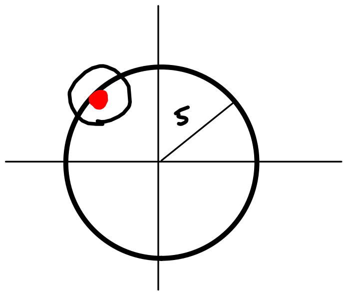
$$= \cos(5x) \cdot 5 = 5\cos(5x)$$

$$\frac{d}{dx}\left[\left[\tan\left((x^5+4x)^{11}\right)\right]^{20}\right]$$

$$= 20\left(\tan\left((x^5+4x)^{11}\right)\right)^{19}\sec^2\left((x^5+4x)^{11}\right)\cdot 11(x^5+4x)^{10}\cdot(5x^4+4)$$

Implicit Differentiation

Equations with graphs that aren't functions



Equation:

$$x^2 + y^2 = 5^2 \implies$$

$$y^2 = 25 - x^2$$

$$y = \pm\sqrt{25 - x^2}$$

$$y = \sqrt{25 - x^2}$$

$$y = -\sqrt{25 - x^2}$$

Top half: $y = \sqrt{25 - x^2} = (25 - x^2)^{\frac{1}{2}} \implies$

$$\frac{dy}{dx} = y' = \frac{1}{2}(25 - x^2)^{-\frac{1}{2}}(-2x) = -\frac{x}{\sqrt{25 - x^2}} = y'$$

Locally, we have *y* as a function of *x*.

Assume y is an implicit function of x.

$$\frac{d}{dx}\left[x^2 + y^2 = 25\right]$$

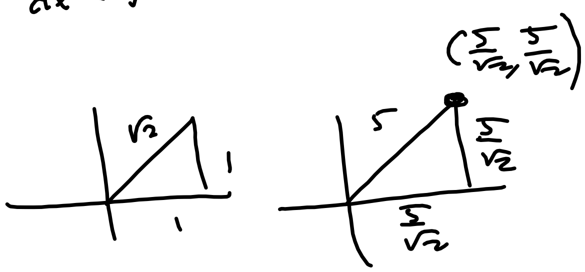
$$= 2x + 2y \cdot \frac{dy}{dx} = 0$$

$$= 2x + 2yy' = 0$$

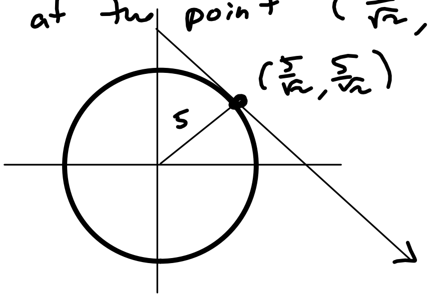
$$\implies 2yy' = -2x$$

$$\implies y' = -\frac{2x}{2y} = -\frac{x}{y}$$

$$\frac{d}{dx}\left[y^2\right] = 2yy'$$



Find an equation of the tangent ~~ta~~ line to the circle at the point $\left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$



$y = -1\left(x - \frac{5}{\sqrt{2}}\right) + \frac{5}{\sqrt{2}}$ END Goal.

$= m(x - x_1) + y_1 = \underline{f'(x_1)(x - x_1) + y_1}$ ↳ Tangent Line.

Implicit: $m = -\frac{x}{y} = -\frac{\frac{5}{\sqrt{2}}}{\frac{5}{\sqrt{2}}} = -1 \Longrightarrow$

$L(x) =$ tangent line $=$ linearization

$= -1\left(x - \frac{5}{\sqrt{2}}\right) + \frac{5}{\sqrt{2}}.$

OLD-SCHOOL:

$f(x) = y = \sqrt{25 - x^2} = (25 - x^2)^{\frac{1}{2}}$

$\Longrightarrow f'(x) = \frac{dy}{dx} = y' = \frac{-x}{\sqrt{25 - x^2}}$

$f'\left(\frac{5}{\sqrt{2}}\right) = \frac{-\frac{5}{\sqrt{2}}}{\sqrt{25 - \left(\frac{5}{\sqrt{2}}\right)^2}} = \frac{-\frac{5}{\sqrt{2}}}{\sqrt{25 - \frac{25}{2}}} = \frac{-\frac{5}{\sqrt{2}}}{\sqrt{\frac{25}{2}}} = \frac{-\frac{5}{\sqrt{2}}}{\frac{5}{\sqrt{2}}} = -1$

$\Longrightarrow \boxed{L(x) = -1\left(x - \frac{5}{\sqrt{2}}\right) + \frac{5}{\sqrt{2}}}$

Circles are "special." You can actually solve for*y*.

$$\frac{d}{dx}\left[x^2 + 3\underset{fg}{x^2y^3} + y^3 = \sin(xy)\right]$$

$$= 2x + 3\left[\underset{f'g}{2xy^3} + \underset{f}{x^2}\cdot \underbrace{3y^2y'}_{g'}\right] + 3y^2y' \qquad = \cos(xy)\left[y + xy'\right]$$

$$\frac{d}{dx}\left[xy\right] = 1y + x\cdot y'$$

Now solve for y.

$$2x + 6xy^3 + 9x^2y^2y' + 3y^2y' = y\cos(xy) + xy'\cos(xy)$$

$$\Rightarrow 9x^2y^2y' + 3y^2y' - x\cos(xy)\cdot y' = y\cos(xy) - 2x - 6xy^3$$

$$\Rightarrow \left(9x^2y^2 + 3y^2 - x\cos(x)\right)y' = y\cos(xy) - 2x - 6xy^3$$

$$y' = \frac{y\cos(xy) - 2x - 6xy^3}{9x^2y^2 + 3y^2 - x\cos(xy)}$$