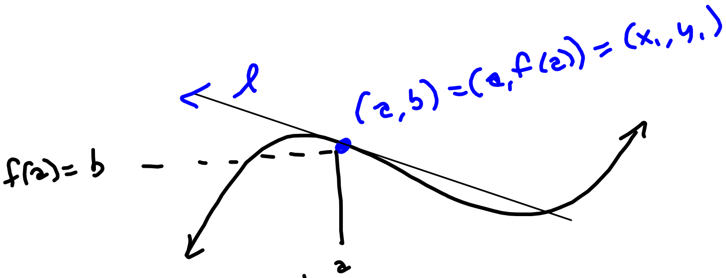
Tangent line at (a, b) has the slope of *f* at *a* .

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{y-b}{x-a} = f'(a)$$

$$\left.\begin{array}{ll} \text{meh} & y-b = f'(a)(x-a) \\ \text{Yeah!} & y = f'(a)(x-a)+b \end{array}\right\} \text{Point-Slope}$$

$$= f'(a)(x-a)+f(a)$$



ℓ = tangent line

$$y - y_1 = m(x-x_1)$$

$$y = m(x-x_1)+y_1$$

$$y = f'(a)(x-a)+f(a) = f(a)+f'(a)(x-a)$$

$$y = f'(3)(x-3)+2$$

Find eqn of line thru $(3,7)$ with slope $m = 11$

$$y = 11(x-3)+7$$

$$g(x)=\begin{cases} x\sin\left(\frac{\pi}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0\end{cases}$$

$$\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} = \frac{h\sin\left(\frac{\pi}{h}\right)-0}{h}$$ Want this @ $x=0$ as $h\to 0$

is far away from $x=0$

$$=\sin\left(\frac{\pi}{h}\right)\xrightarrow{h\to 0}\nexists$$

#6 on Week 4 is a LITTLE advanced, but it doesn't QUITE require you to know what the derivative of sine is in order to answer the question.

The idea is that we're trying to find $g'(0)$, to see if it exists.

$$\lim_{h\to 0}\frac{g(0+h)-g(0)}{h} = \lim_{h\to 0}\frac{h\sin\left(\frac{\pi}{h}\right)-0}{h} = \lim_{h\to 0}\left(\sin\left(\frac{\pi}{h}\right)\right)$$ which

$\nexists$ and that's pretty much it.

$f(x) = x^2 + 5x - 6$

$f'(2) = \lim_{h \to 0} \frac{f'(2+h) - f(2)}{h}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Find $f'(2)$ by plugging in $x = 2$ @ the end.
(after you've computed the limit).

$f(x) = \sqrt{x}$

$(a-b)(a+b)$

$$\frac{f(x+h)-f(x)}{h} = \left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$a^2 - b^2$

$$= \frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \frac{1}{\sqrt{x+h}+\sqrt{x}} \xrightarrow{h \to 0} \frac{1}{\sqrt{x}+\sqrt{x}}$$

$(h \neq 0)$

$$= \frac{1}{2\sqrt{x}}$$

$$f(x) = \frac{1}{x}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{1}{x+h} \cdot \frac{x}{x} - \frac{1}{x} \cdot \frac{x+h}{x+h}}{h}$$

$$= \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \frac{1}{h}\left[\frac{x-x-h}{x(x+h)}\right] = \frac{1}{h}\left[\frac{-h}{x(x+h}\right]$$

$$= \frac{-1}{x(x+h)} \xrightarrow{h \to 0} \frac{-1}{x(x+0)} = \boxed{-\frac{1}{x^2} = f'(x)}$$

$h \neq 0$

Recall Limit Laws: If $\lim f, \lim g$ exist and $a, b \in \mathbb{R}$

$\lim(f \pm g) = \lim f \pm \lim g$

$\lim(af) = a \lim f$

$\lim(fg) = \lim f \lim g$

$\lim\left(\frac{f}{g}\right) = \frac{\lim f}{\lim g}$ provided $\lim g \neq 0$

$\lim(af + bg) = a \lim f + b \lim g$ (LINEAR OPERATOR)

Derivatives are limits!

The derivative operator is Linear

$f'(x) = \frac{df}{dx}$

$\frac{d}{dx}[af + bg] = a\frac{df}{dx} + b\frac{dg}{dx}$ LINEAR OPERATOR.

$\frac{d}{dx^2}[5x^2 - 3x] = 5\frac{d}{dx}[x^2] - 3\frac{d}{dx}[x]$

$= 5[2x] + 3[1]$

$= 10x + 3$