Selected Questions from Fall '24 Midterm.

1. (5 pts each) Evaluate the following limits, if they exist. If one does not exist, explain why.

a. $\lim\limits_{x \to 2^-} \dfrac{x^2+5x-14}{|x-2|}$ b. $\lim\limits_{x \to 2^+} \dfrac{x^2+5x-14}{|x-2|}$ c. $\lim\limits_{x \to 2} \dfrac{x^2+5x-14}{|x-2|}$

Let's talk about absolute value.

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$|3| = 3$

$|-3| = -(-3) = 3$

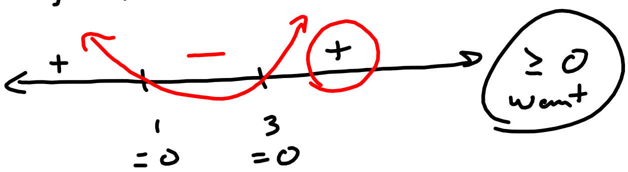
$$|x-2| = \begin{cases} x-2 & \text{if } x-2 \geq 0 \\ -(x-2) & \text{if } x-2 < 0 \end{cases} = \begin{cases} x-2 & \text{if } x \geq 2 \\ -x+2 & \text{if } x < 2 \end{cases}$$

$$g(x) = |x^2-4x+3| = \begin{cases} x^2-4x+3 & \text{if } x^2-4x+3 \geq 0 \\ -(x-4x+3) & \text{if } x^2-4x+3 < 0 \end{cases}$$

"1"

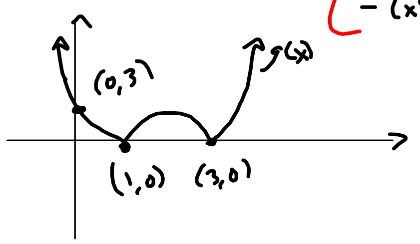
$x^2-4x+3 \geq 0$

$(x-3)(x-1) \geq 0$ It's an x^2 End-Behavior Graphic



SIGN PATTERNS

$$g(x) = |x^2-4x+3| = \begin{cases} x^2-4x+3 & \text{if } -\infty < x \leq 1 \text{ OR } 3 \leq x < \infty \\ -(x^2-4x+3) & \text{if } 1 < x < 3 \end{cases}$$

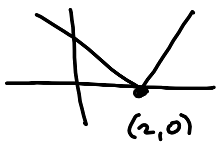


(a) $\lim\limits_{x\to 2^-} \dfrac{x^2+5x-14}{|x-2|} = \lim\limits_{x\to 2^-} \dfrac{(x+7)\cancel{(x-2)}}{-\cancel{(x-2)}} = \cancel{\lim\limits_{x\to 2^-} (x+7)} =$

-1 downstairs

No.

$= \lim\limits_{x\to 2^-} \dfrac{x+7}{-1} = \boxed{-9 = \lim\limits_{x\to 2^-} f}$



(b) $\lim\limits_{x\to 2^+} \dfrac{x^2+5x-14}{|x-2|} = \lim\limits_{x\to 2^+} \dfrac{(x+7)\cancel{(x-2)}}{\cancel{(x-2)}} = \lim\limits_{x\to 2^+} (x+7) = \boxed{9 = \lim\limits_{x\to 2^+} f}$

(c) $\lim\limits_{x\to 2} \dfrac{x^2+5x-14}{|x-2|} \quad \nexists\,!$

Why? B/C $\lim\limits_{x\to 2^-} f = -9 \neq 9 = \lim\limits_{x\to 2^+} f$

Other ways of "doing" x^2-4x+3:

Complete the Square:

$$x^2-4x+3 = x^2-4x+2^2-4+3 = (x^2-4x+2^2)-1$$

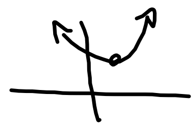
$$= (x-2)^2-1 \overset{\text{SET}}{=} 0 \Longrightarrow$$

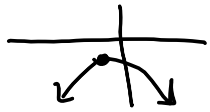
$$(x-2)^2 = 1 \qquad \sqrt{(x-2)^2} =$$

skip? $\left\{ \begin{array}{l} \sqrt{(x-2)^2} = \sqrt{1} \\ |x-2| = 1 \end{array} \right.$ $\qquad \sqrt{3^2} = 3 \qquad \sqrt{(-3)^2} = 3$

$$x-2 = \pm 1$$

$$x = 2 \pm 1 \nearrow 3, \searrow 1$$





Negative Discriminant pictures

$a=1,\ b=-4,\ c=3$

$$b^2-4ac = 4^2-4(1)(3) = 16-12 = 4 = \text{Discriminant}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-4) \pm \sqrt{4}}{2(1)} = \frac{4 \pm 2}{2} \nearrow \frac{6}{2} = 3, \searrow \frac{2}{2} = 1$$

ORIENTATION PAGE FOR COLLEGE ALGEBRA

LOCKDOWN BROWSER

2. Consider the piecewise-defined function $f(x)=\begin{cases} x^2-4x+5 & \text{if } x<3 \\ -\frac{1}{2}x+\frac{7}{2} & \text{if } x\geq 3 \end{cases}$.

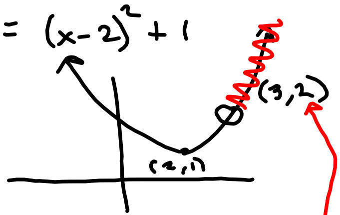
   a. (5 pts) Sketch the graph of $f(x)$. Label the x- and y-intercepts, the suture point(s), and the vertex of the quadratic piece, if it's in the picture. When I say "Label," I mean an ordered pair, like (0, 5), next to the point.

   b. (5 pts) On what interval(s) is $f(x)$ continuous? Explain.

Suture Point: $x=3$

$x^2-4x+5 = \underbrace{x^2-4x+2^2}-4+5$ -

$= (x-2)^2+1$



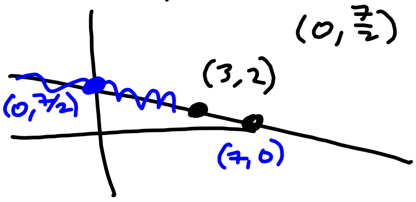
$3^2-4(3)+5 = 2$

$-\frac{1}{2}x+\frac{7}{2}$ SET 0 →

$-x+7=0$ →

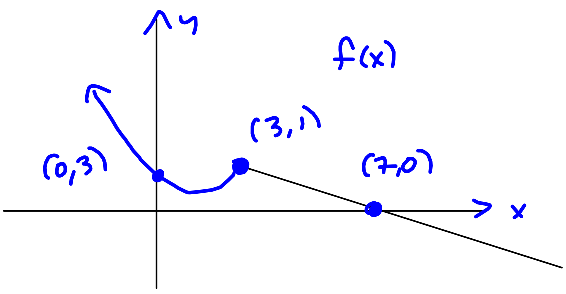
$x=7$ (7,0)

$(0,\frac{7}{2})$



$-\frac{1}{2}(3)+\frac{7}{2} = \frac{-3+7}{2} = \frac{4}{2} = 2$

Combine:



f is cnt^s. f cnt^s on $(-\infty, 3)$ from x^2-4x+3
" " " $(3,\infty)$ " $-\frac{1}{2}x+\frac{7}{2}$

And $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = 1$

So they agree @ the same point.

f's cnt^s on each piece, b/c they're polynomials

And $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = 1$

So they agree @ the same point.

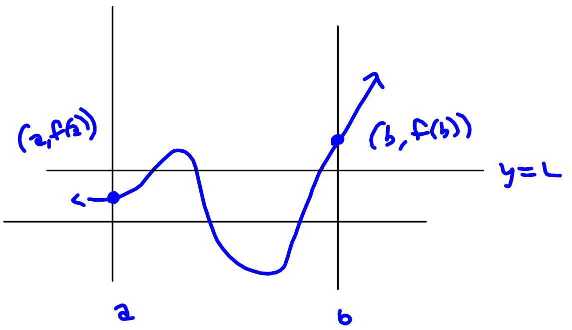
Intermediate Value Theorem

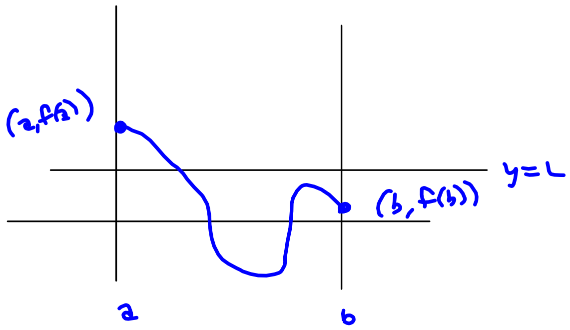
If *f* is continuous on   and there's a number

f cnt^s on $[a,b]$

and $f(a) < L < f(b)$ OR $f(a) > L > f(b)$ $\Longrightarrow$

$f(c) = L$ for some $c \in (a,b)$.





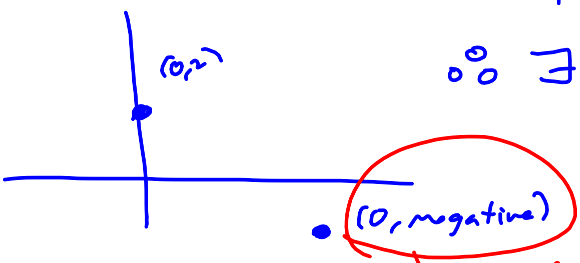
IVT does NOT tell you how to find *c*. It just says there is one!

7. (5 pts) Prove that the equation $f(x) = x^2 - 4x\sin(x) + 2$ has a root in the interval $(0,2)$, but *do not solve*!

f is cnt's. $f(0) = 2 > 0$, $f(2) = 4 - 16\sin(2) + 2 \approx -1.27437941461 < 0$

Radians!

$\therefore \exists\, c \in (0,2) \ni f(c) = 0$



No. (2, negative) $\approx (2, -1.27441)$

Yes, Steve. The surface of a sphere is an elliptical space. This means there are FEWER parallel lines, not MORE. The point is that I can travel around the universe to cross the road without crossing the center line, technically. (Contracting space)

But we're living in 2-dimensional Euclidean space for the purposes of these discussions. (The plane). (10th- or 11th-grade geometry).

The other kind of space is hyperbolic space, where there are MORE parallel lines. Even lines that seem to NOT be parallel can curve away from each other without touching. (Expanding Space)