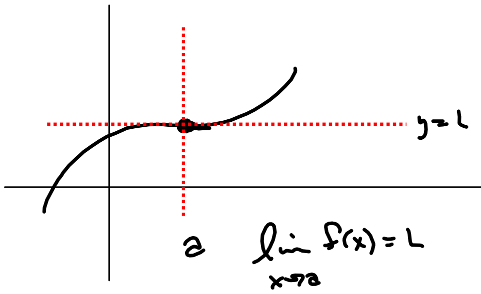
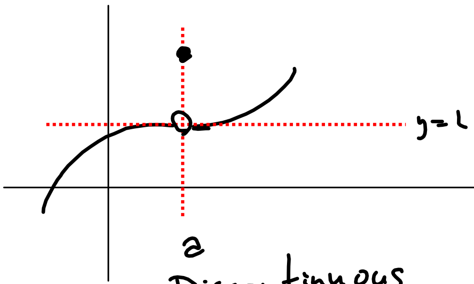
Limit Exists $\lim_{x \to a} f(x)$



$\lim_{x \to a} f(x) = L$

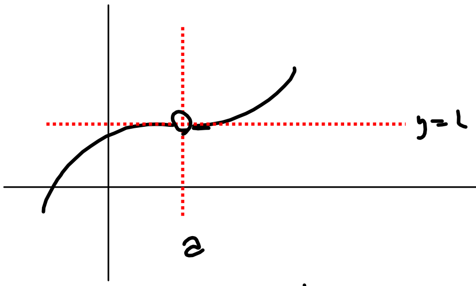
Continuous

$\lim_{x \to a} f(x) = L = f(a)$



Discontinuous

$\lim_{x \to a} f(x) = L \neq f(a)$



Discontinuous

$\lim_{x \to a} f(x) = L$, but $f(a) \nexists$!

$f(x) = x^2 + 3x - 2$

$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c} = \lim_{x \to c} \frac{x^2 + 3x - 2 - (c^2 + 3c - 2)}{x - c}$$

$$= \lim_{x \to c} \frac{x^2 + 3x - 2 - c^2 - 3c + 2}{x - c} = \lim_{x \to c} \frac{x^2 - c^2 + 3x - 3c}{x - c}$$

$$= \lim_{x \to c} \frac{(x-c)(x+c) + 3(x-c)}{x - c} = \lim_{x \to c} \frac{(x-c)[x + c + 3]}{x - c} = \lim_{x \to c} [x + c + 3]$$

$$= c + c + 3 = 2c + 3 = \text{“} f'(c) \text{”}$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2+3(x+h)-2-(x^2+3x-2)}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+\boxed{3x}+3h-2-x^2-\boxed{3x}+2}{h} = \lim_{h\to 0} \frac{2xh+3h+h^2}{h}$$

$$= \lim \frac{h(2x+3+h)}{h} = \lim_{h\to 0} (2x+3+h) = 2x+3 = \text{"}f'(x)\text{"}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+3(x+h)-2-(x^2+3x-2)}{h}$$

$$= \frac{x^2+2xh+h^2+3x+3h-2-x^2-3x+2}{h} = \frac{2xh+h^2+3h}{h}$$

$$= \frac{h(2x+h+3)}{h} = \underset{h\neq 0}{2x+h+3} \xrightarrow{h\to 0} 2x+3 = \text{"}f'(x).\text{"}$$