solve x^3+x+3=5.3

NATURAL LANGUAGE | MATH INPUT

Input interpretation

solve	$x^3 + x + 3 = 5.3$

Results

$x \approx 1.0711$

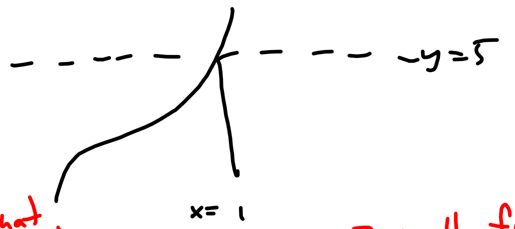
solve x^3+x+3=4.7

NATURAL LANGUAGE | MATH INPUT

Input interpretation

solve	$x^3 + x + 3 = 4.7$

For 1.7 #19



$$\lim_{x\to 1} (x^3+x+3) = 5$$

ε-δ Formulation :

such that
so that

Want $\delta > 0$ $\exists$ $0 < |x-1| < \delta \longrightarrow$

$|f(x) - L| = |x^3+x+3-5| < \varepsilon$ $\forall$ $\varepsilon > 0$.

For all, for each,
for every.

So we want

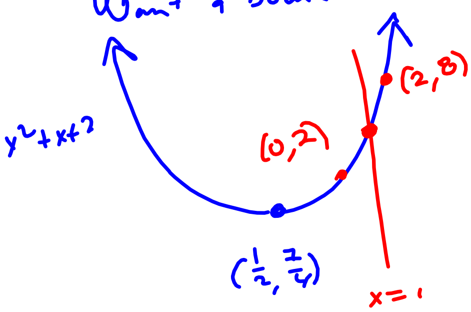
$|x^3+x-2| < \varepsilon$

$$\underbrace{|x-1|}_{<\delta}\underbrace{|x^2+x+2|}_{\text{Need a bound on this.}} < \varepsilon$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

$x^2+x+2 = x^2+x+\left(\frac{1}{2}\right)^2 - \frac{1}{4} + \frac{8}{4} = \left(x+\frac{1}{2}\right)^2+\frac{7}{4}$

Want a bound on x^2+x+2 close to $x=1$



Assume $\delta \le 1$

$\therefore \ |x^2+x-2| \le 8$ for $x \in [0,2]$

Now: $|x-1||x^2+x+2| < \delta \cdot 8 < \varepsilon \Rightarrow \delta < \frac{\varepsilon}{8}$

Define $\delta = \min\{1, \frac{\varepsilon}{8}\}$

Claim: $\lim_{x\to 1}(x^3+x+3) = 5$

<u>Proof</u>: Let $\varepsilon > 0$ be given.

Define $\delta = \min\{1, \frac{\varepsilon}{8}\}$. Then $0 < |x-1| < \delta$ implies

$|f(x)-L| = |x^3+x+3-5| = |x^3+x-2| = |x-1||x^2+x+2|$

$< \delta \cdot 8 \le \frac{\varepsilon}{8} \cdot 8 = \varepsilon$ ■

The strategy for finding the Largest δ possible

$\exists \qquad 5-\varepsilon < x^3+x+3 < 5+\varepsilon$

$-\varepsilon < x^3+x-2 < \varepsilon$ Means $x^3+x-2 > -\varepsilon$

AND

$\underbrace{x^3+x-2 < \varepsilon}$

This would yield the smaller δ, hence the δ you want.

$|x^3+x-2| < \varepsilon$

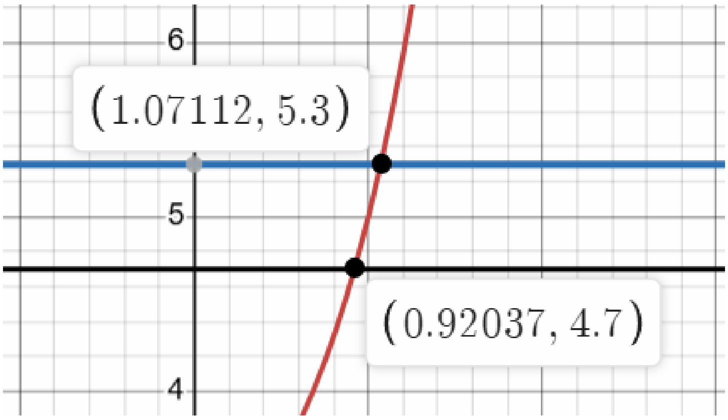
Kind of a dumb exercise.

$\varepsilon = .3$, solve $x^3+x+3 = 5.3$

$x^3+x+3 = 4.7$

Subtract Answer from 1. See previous page.

The smaller of the 2 δ's is the largest δ that will work for both.


6
(1.07112, 5.3)
5
(0.92037, 4.7)
4