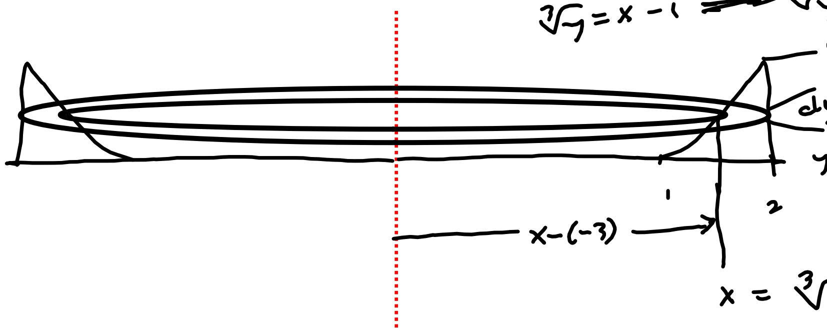
Week 14 $y=(x-1)^3$ from $x=1$ to $x=2$ about $x=-3$

solutions are messed up...

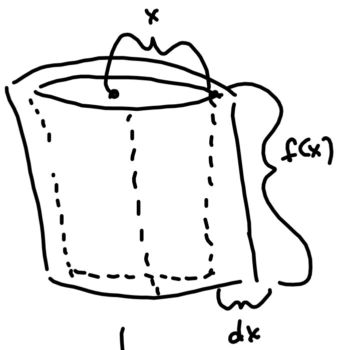
$y=(x-1)^3$

$\sqrt[3]{y}=x-1 \Longrightarrow \sqrt[3]{y}+1=x$

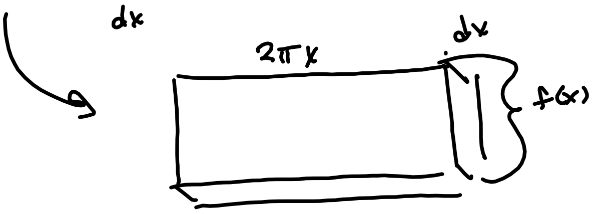


$$\pi\int_{y=0}^{y=1}(\text{outer}^2-\text{inner}^2)\,dy$$

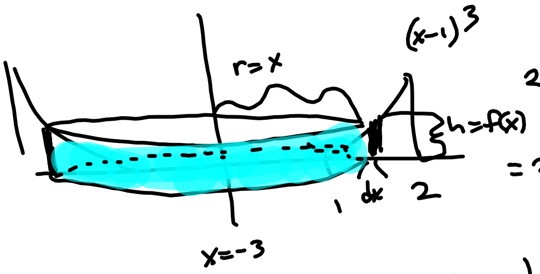
$$\text{Volume} = \pi\int_0^1\left((2-(-3))^2-(\sqrt[3]{y}+1-(-3))^2\right)dy$$



$\text{Volume} = \pi\int$



Shell Method: Revolve about y-axis



$$2\pi\int_1^2 x f(x)\,dx$$

$$=2\pi\int_1^2 x(x-1)^3\,dx$$

$2\pi r h\,dx$

$= 2\pi x f(x)dx$ Volume of cylindrical shell, with height $f(x)$, radius x & thickness $\Delta x = dx$.

$f(x) = x^2+5x+11$

$= x^2+5x+\left(\frac{5}{2}\right)^2-\frac{25}{4}+\frac{44}{4}$

$= \left(x+\frac{5}{2}\right)^2 + \frac{19}{4} = f(x)$

To find the inverse, solve

$\left(x+\frac{5}{2}\right)^2+\frac{19}{4}=y$ for x

$\left(x+\frac{5}{2}\right)^2 = y - \frac{19}{4}$

$x+\frac{5}{2} = \pm\sqrt{\frac{4y-19}{4}} = \pm\frac{\sqrt{4y-19}}{2}$

$y = \frac{-5\pm\sqrt{4y-19}}{2}$

Don't like completing the square?

$x^2+5x+11=y$

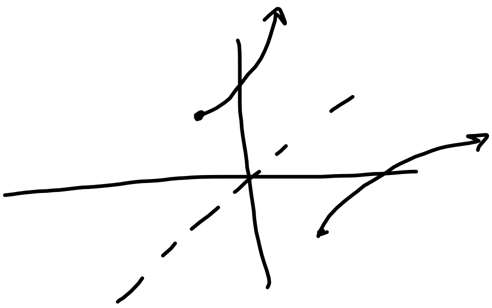
$x^2+5x+(11-y)=0$

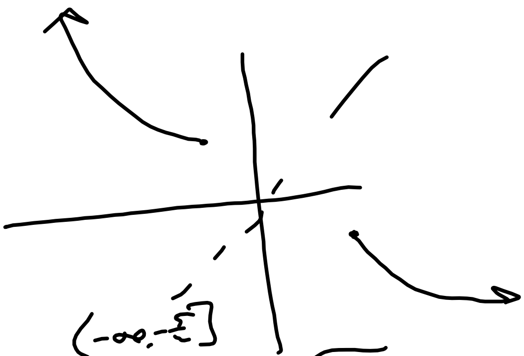
$a=1,\ b=5,\ c=11-y$

$b^2-4ac = 5^2-4(1)(11-y) = 25-44+4y = 4y-19$

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-5\pm\sqrt{4y-19}}{2} = x$

$\Rightarrow f^{-1}(x) = \frac{-5+\sqrt{4y-19}}{2}$





$\Rightarrow f^{-1}(x) = \frac{-5-\sqrt{4y-19}}{2}$

Sledgehammer for factoring quadratics

$$360x^2 - 1617x - 3146$$

$a=360,\ b=-1617,\ c=-3146$

$b^2-4ac = 1617^2 - 4(360)(-3146) = 7144929$

$$x = \frac{1617 \pm \sqrt{7144929}}{2(360)} = \frac{1617 \pm 2673}{720} \to -\frac{22}{15},\ \frac{143}{24}$$

$$360\left(x+\frac{22}{15}\right)\left(x-\frac{143}{24}\right)$$

$$= 15\cdot 24\left(x-\frac{22}{15}\right)\left(x-\frac{143}{24}\right)$$

$$= (15x-22)(24x-143)$$

$$\begin{array}{r} 15 \\ \times\ 24 \\ \hline 60 \\ 300 \\ \hline 360 \end{array}$$

Differentiate

$$\frac{d}{dx}\left[5\cdot 7^{2x-3}\right]$$

STOP!

$$= 5\frac{d}{dx}\left[7^{2x-3}\right] = \boxed{5\ln(7)\cdot 7^{2x-3}\cdot(2)}$$

$$= 10\ln(7)\cdot 7^{2x-3}$$

I always remember:

$$\frac{d}{dx}\left[e^x\right] = e^x$$

e^x & $\ln(x)$ are inverses

$$\int e^x dx = e^x + C$$

$$e^{\ln(x)} = \ln(e^x) = x$$

$$\frac{d}{dx}\left[7^x\right]$$

$$7 = e^{\ln(7)} = \ln(e^7)$$

$$\frac{d}{dx}\left[\left(e^{\ln(7)}\right)^x\right] = \frac{d}{dx}\left[e^{x\ln(7)}\right] = \frac{d}{dx}\left[e^{(\ln(7))x}\right]$$

$$= \left(e^{(\ln(7))x}\right)\cdot\ln(7)$$

$$= \left(e^{\ln(7)}\right)^x\cdot\ln(7)$$

$$= 7^x\cdot\ln(7)$$

$$\frac{d}{dx}\left[\log_3(x)\right] = \frac{d}{dx}\left[\frac{\ln(x)}{\ln(3)}\right] = \frac{1}{\ln(3)}\frac{d}{dx}\left[\ln(x)\right]$$

$$\frac{1}{\ln(3)}\cdot\frac{1}{x}$$

$$\frac{d}{dx}\left[e^x\right] = e^x,\quad \frac{d}{dx}\left[\ln(x)\right]$$

Properties of Logs:

$$\log(ab) = \log(a) + \log(b)$$

$$\log(x^y) = y\log(x)$$

$$\log\left(\frac{a}{b}\right) = \log\left(a\cdot\frac{1}{b}\right)$$
$$= \log(a) + \log\left(\frac{1}{b}\right)$$
$$= \log(a) + \log(b^{-1})$$
$$= \log(a) - \log(b)$$

Differentiate:

$$y = \frac{\left(\sqrt[5]{x^2+\sin(x)}\right)^{23}}{\left(\cos^2(x)\right)\left(x^3-7x\right)^{17}}$$

$$\ln(y) = \ln\left(\left(x^2+\sin(x)\right)^{23/5}\right) - \ln\left(\cos^2(x)\right) - \ln\left(\left(x^3-7x\right)^{17}\right)$$

$$= \frac{23}{5}\ln\left(x^2+\sin(x)\right) - 2\ln\left(\cos(x)\right) - 17\ln\left(x^3-7x\right)$$

$$\frac{d}{dx}\left[\ln(y)\right] = \frac{d}{dx}\left[\text{ mess }\right]$$

$$\frac{y'}{y} = \frac{23}{5}\left(\frac{2x+\cos(x)}{x^2+\sin(x)}\right) - 2\left(\frac{-\sin(x)}{\cos(x)}\right) - 17\left(\frac{3x^2-7}{x^3-7x}\right)$$

$$y' = \left[\frac{23}{5}\left(\frac{2x+\cos(x)}{x^2+\sin(x)}\right) - 2\left(\frac{-\sin(x)}{\cos(x)}\right) - 17\left(\frac{3x^2-7}{x^3-7x}\right)\right]\left(\frac{\left(\sqrt[5]{x^2+\sin(x)}\right)^{23}}{\left(\cos^2(x)\right)\left(x^3-7x\right)^{17}}\right)$$