§5.1?

§5.2?

§6.1 - 6.4

(5 pts) Let $f(x) = 2x^3 - 7x^2 + 5x$ and $g(x) = x^3 - x^2 - 3x$. Find the area of the region bounded by $f(x), g(x), x = 0,$ and $x = 5$.

$f(x) = x(2x^2 - 7x + 5) = x(2x-5)(x-1)$

$g(x) = x(x^2 - x - 3) =$

$$\int_0^5 |f(x) - g(x)|\,dx = \int_0^5 |x^3 - 6x^2 + 8x|\,dx = \int_0^1 |x(x-4)(x-2)|\,dx$$

($\underset{\substack{0\\=0}}{-\;|}\;+\;\underset{\substack{2\\=0}}{|}\;-\;\underset{\substack{4\\=0}}{|}\;+\;\rightarrow$ $\quad x(x-4)(x-2)$ SIGN PATTERNS)

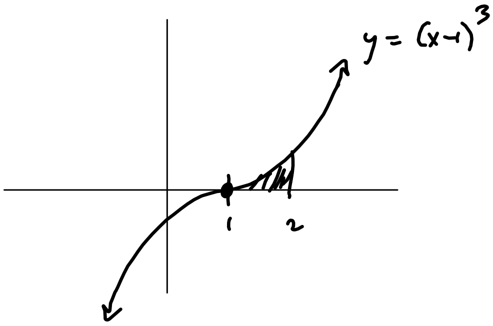
$$= \int_0^2 (x^3 - 6x^2 + 8x)\,dx - \int_2^4 (x^3 - 6x^2 + 8x)\,dx + \int_4^5 (x^3 - 6x^2 + 8x)\,dx$$

$$= \left.\frac{x^4}{4} - 2x^3 + 4x^2\right]_0^2 - \left[\frac{x^4}{4} - 2x^3 + 4x^2\right]_2^4 + \left[\frac{x^4}{4} - 2x^3 + 4x^2\right]_4^5$$

2. Consider the region bounded by $y=(x-1)^3$, $y=0$, $x=1$, and $x=2$.

   a. (5 pts) Sketch this region and find its area.

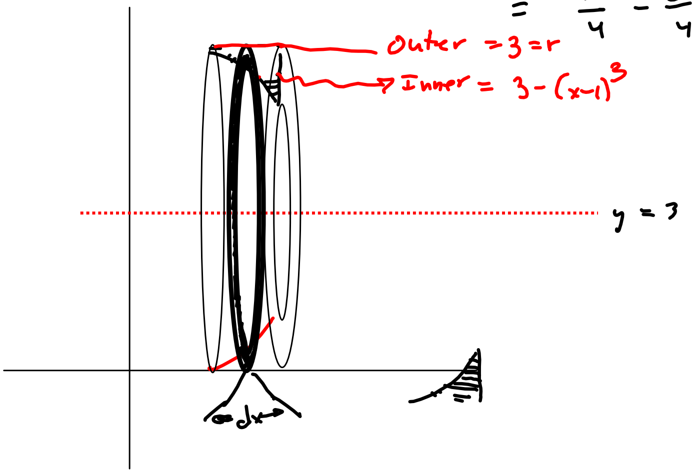
   b. (5 pts) Sketch the solid obtained by rotating this region about the line $y=3$. Include a representative washer on your graph and write the integral for finding its volume by the disk method.



$$\text{Area} = \int_1^2 (x-1)^3\,dx$$

$$= \left.\frac{(x-1)^4}{4}\right]_1^2$$

$$= \frac{1^4}{4} - \frac{0^4}{4} = \boxed{\frac{1}{4} = \text{Area}}$$



$V = \pi r^2 h$ = Volume of a disk of thickness h, radius r

Outer Disk - Inner Disk = Washer

Volume of the Solid of Revolution

is Volume of outer - Volume of Inner.

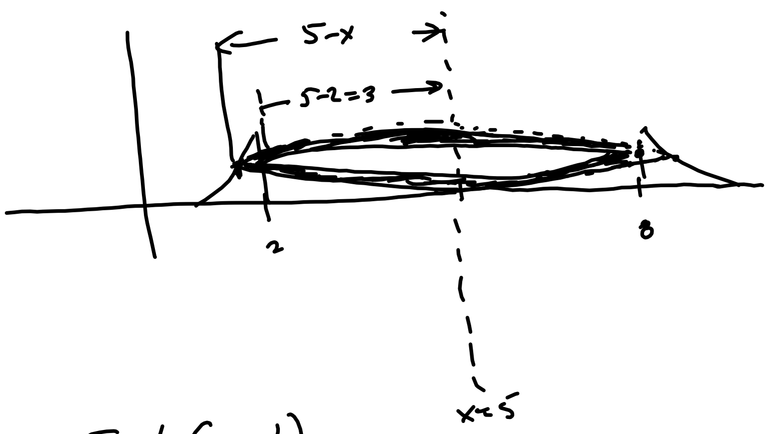
$$\int_1^2 \pi \cdot 3^2 \cdot dx \; - \; \int_1^2 \pi \cdot (x-1)^3\,dx$$

$$= \pi \int_a^b (\text{outer}^2 - \text{inner}^2)\,dx$$

$$= \pi \int_a^b (y_o^2 - y_i^2)\,dx$$

$$= \pi \int_1^2 \left(3^2 - \left((x-1)^3\right)^2\right)dx \text{, where } y = f(x)$$

Revolve $y=(x-1)^3$, $x=1$, $x=2$, $y=0$ about $x=5$



Find (need)

$x = g(y)$

$y = (x-1)^3 \longrightarrow \sqrt[3]{y} = x-1 \longrightarrow$

$x = \sqrt[3]{y} + 1$

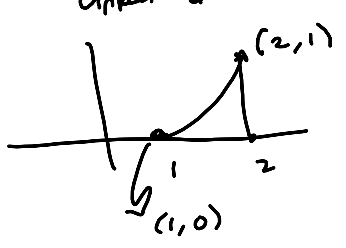
$$\text{Volume} = \pi \int_c^d x^2 \, dy \quad \text{, where } x = g(y)$$

outer $5-x = 5-(\sqrt[3]{y}+1)$

Inner $= 3$ (from $5-2$)

$$\pi \int_0^1 \left((\sqrt[3]{y}+1)^2 - 3^2 \right) dx$$

upper & lower limits from graph



$f(x) = x^2+5x+11$ on $[-\frac{5}{2},\infty)$

$$x^2+5x+11 = y$$

$$x^2+5x+\left(\tfrac{5}{2}\right)^2 = y-11+\frac{25}{4} = y+\frac{-44+25}{4} = y-\frac{19}{4}$$

$$\left(x+\tfrac{5}{2}\right)^2 = y-\frac{19}{4}$$

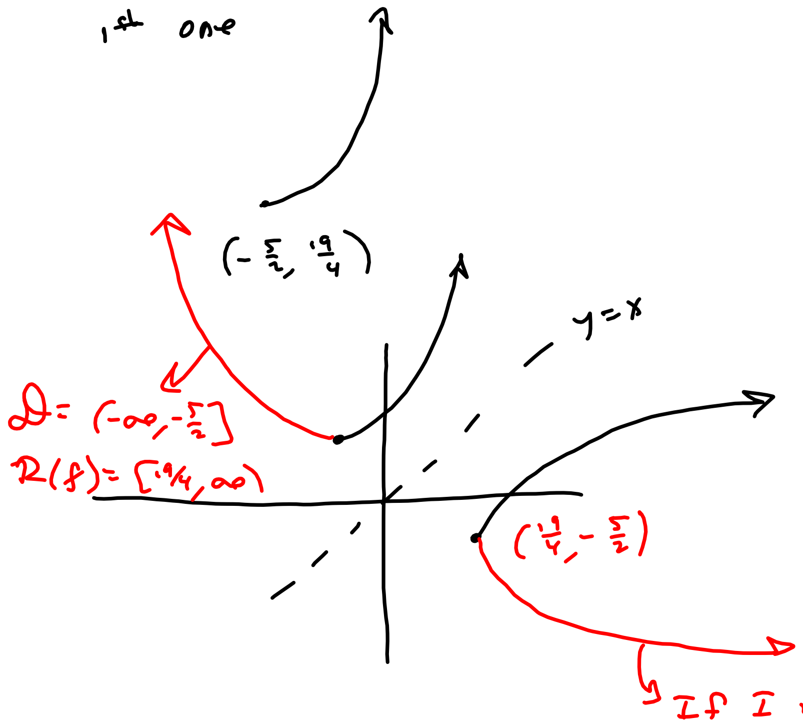
$$x+\frac{5}{2} = \pm\sqrt{y-\frac{19}{4}} = \pm\sqrt{\frac{4y-19}{4}} = \pm\frac{\sqrt{4y-19}}{2}$$

$$x = \frac{-5\pm\sqrt{4y-19}}{2}$$ want "+"

$$f^{-1}(x) = \frac{-5+\sqrt{4x-19}}{2}$$

What about if we define/restrict the original domain to $(-\infty,-\frac{5}{2}]$?

$$x^2+5x+11$$
$$= \left(x^2+5x+\left(\tfrac{5}{2}\right)^2\right)-\frac{25}{4}+\frac{44}{4}$$
$$= \left(x+\tfrac{5}{2}\right)^2+\frac{19}{4}$$



If I restrict to $(-\infty,-\frac{5}{2}]$ on $f(x)$

$\mathcal{D}(f) = \mathcal{R}(f^{-1}) = [-\infty,-\frac{5}{2}]$

$\mathcal{D}(f^{-1}) = \mathcal{R}(f)$!