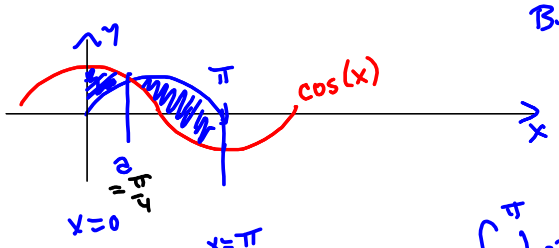
Section 5.1: Area Between two curves is the integral of their difference.

Need to know where they intersect, so you can write the right integral.

Find the area bounded by $\sin(x)$ & $\cos(x)$ on $[0,\pi]$



Better-posed:
Find area bounded by $\sin(x)$, $\cos(x)$, $x=0$, $x=\pi$

$$\text{Area} = \int_0^{\pi} |\sin(x) - \cos(x)|\, dx$$

$$|\sin(x) - \cos(x)| = \begin{cases} \sin(x) - \cos(x) & \text{if } \sin(x)-\cos(x) \geq 0 \\ -(\sin(x) - \cos(x)) & \text{if } \sin(x) - \cos(x) < 0 \end{cases}$$

on $[0,\pi]$: By inspection of the graph, we see it's
positive when $a < x < \pi$
& negative " $0 \leq x < a$
(Haven't found a yet.

$$\text{Area} = \int_0^a -(\sin(x) - \cos(x))\, dx + \int_a^{\pi} (\sin(x) - \cos(x))\, dx$$

Find a:
$$\sin(x) - \cos(x) = 0$$
$$\sin(x) = \cos(x)$$
$$\frac{\sin(x)}{\cos(x)} = 1$$
$$\tan(x) = 1$$

$\rightarrow x = \frac{\pi}{4} = 45^\circ$

$$\text{Area} = \int_0^{\frac{\pi}{4}} -(\sin(x) - \cos(x))\,dx + \int_{\frac{\pi}{4}}^{\pi} (\sin(x) - \cos(x))\,dx$$

$(-\sin(x) + \cos(x))$

$$= \Big[\cos(x) - \sin(x)\Big]_0^{\frac{\pi}{4}} + \Big[-\cos(x) - \sin(x)\Big]_{\frac{\pi}{4}}^{\pi}$$

$+$

$$= \cos\left(\tfrac{\pi}{4}\right) + \sin\left(\tfrac{\pi}{4}\right) - \Big[\cos(0) + \sin(0)\Big]$$

$$+ \Big[-\cos(\pi) - \sin(\pi)\Big] - \Big[\cos\left(\tfrac{\pi}{4}\right) - \sin\left(\tfrac{\pi}{4}\right)\Big]$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - [1] + \Big[-(-1) + 0\Big] - \Big[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\Big]$$

$$= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2} \approx 2.82842712475$$

Desmos: 2.82842712475

Area between: $\int(\text{upper} - \text{lower})\,dx$

If the upper & lower switch, you have to change signs.

$$\text{Area between } f(x) \text{ \& } g(x) \text{ on } [a,b] = \int_a^b \underbrace{|f(x) - g(x)|}_{\text{upper} - \text{Lower.}}\,dx$$

$f(x) = \text{abs}(\sin(x) - \cos(x))$

$\int_0^{\pi} f(x)\,dx$

$= 2.82842712475$

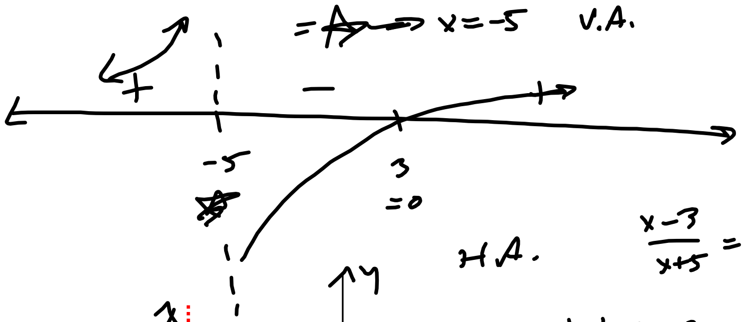
$2\sqrt{2}$

$= 2.82842712475$

$|\sin(x) - \cos(x)|$

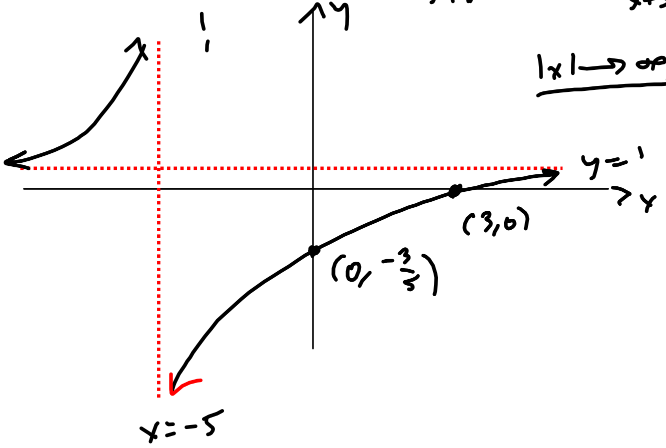
$\frac{x-3}{x+5} = 0 \Rightarrow x=3 \rightsquigarrow (3,0)$ x-int

$= \cancel{A} \Rightarrow x=-5$ V.A.



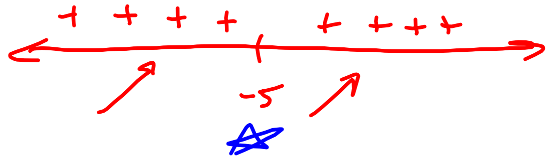
H.A. $\frac{x-3}{x+5} = \frac{x(1-\frac{3}{x})}{x(1+\frac{5}{x})} = \frac{1-\frac{3}{x}}{1+\frac{5}{x}}$

$\underline{|x| \to \infty}$ $\frac{1}{1} = \boxed{y=1 \text{ is H.A.}}$



$R' = \frac{1(x+5)-(x-3)(1)}{(x+5)^2}$

$= \frac{x+5-x+3}{(x+5)^2} = \frac{8}{(x+5)^2}$



$R'' = \frac{d}{dx}\left[8(x+5)^{-2}\right] = -2(8)(x+5)^{-3}(1) = \frac{-16}{(x+5)^3}$

True or False: f & g cnt^s $\Longrightarrow$

$$\int_a^b (f(x)g(x))\,dx = \int_a^b f(x)\,dx \int_a^b g(x)\,dx$$

Write much. Think little.