$$\int_{1/2}^{1} \frac{\cos(x^{-2})}{x^3}\,dx$$

$u = x^{-2}$

$du = -2x^{-3}\,dx \Longrightarrow dx = \frac{du}{-2x^{-3}} = \frac{x^3\,du}{-2}$

$\cancel{u(\tfrac{1}{2}) = \cos((\tfrac{1}{2})^{-2})}$

$\cancel{= \cos(4)}$

$\cancel{u(1) = \cos(1^{-2}) = \cos(1)}$

No

$\cancel{\int_{\cos(4)}^{\cos(1)}}$

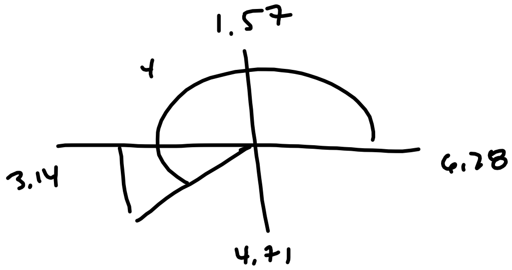
No. $u(\tfrac{1}{2}) = (\tfrac{1}{2})^{-2} = 4$

$u(1) = 1^{-2} = 1$

$$\int_4^1 \frac{\cos(u)\left(\frac{\cancel{x^3}\,du}{-2}\right)}{\cancel{x^3}} = \int_4^1 \frac{\cos(u)\,du}{-2} = -\frac{1}{2}\int_4^1 \cos(u)\,du$$

$$= -\frac{1}{2}\Big[\sin(u)\Big]_4^1 = -\frac{1}{2}\Big[\sin(1) - \sin(4)\Big] = \frac{\sin(1)-\sin(4)}{-2}$$

$$= \frac{\sin(4)-\sin(1)}{2} \approx -0.7991367400$$



$$\int_0^{T/2} \sin(2\pi t/T - \alpha)\, dt = \int_0^{\frac{T}{2}} \sin\left(\frac{2\pi}{T}t - \alpha\right) dt$$

$u = \frac{2\pi}{T}t \Rightarrow du = \frac{2\pi}{T}dt$

$\Rightarrow dt = \frac{T}{2\pi}du$

$u(0) = \frac{2\pi}{T}(0) = 0$

$u\left(\frac{T}{2}\right) = \frac{2\pi}{T}\left(\frac{T}{2}\right) = \pi$

$$= \int_0^{\pi} \sin(u)\frac{T}{2\pi}du = \frac{T}{2\pi}\int_0^{\pi} \sin(u)du$$

$$= \frac{T}{2\pi}\Big[-\cos(u)\Big]_0^{\pi}$$

$$= \frac{T}{2\pi}\Big[-\cos(\pi) - (-\cos(0))\Big] = \frac{T}{2\pi}\Big[-(-1) - (-1)\Big]$$

$$= \frac{T}{2\pi}\Big[1+1\Big] \boxed{= \frac{T}{\pi}}$$

$$\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$$

$u = 1+\sqrt{x} = 1 + x^{\frac{1}{2}}$

$\Rightarrow du = \frac{1}{2} x^{-\frac{1}{2}} dx \Rightarrow$

$dx = 2x^{\frac{1}{2}} du = 2\sqrt{x}\, du$

$$\boxed{\begin{aligned} u(0) &= 1+\sqrt{0} = 1 \\ u(1) &= 1+\sqrt{1} = 2 \end{aligned}} \rightarrow \text{Sometimes this can be painful.}$$

What if ↑ is painful? Just leave the limits in terms of x

$$= \int_{x=0}^{x=1} \frac{2\sqrt{x}\, du}{u^4}$$

What to do about the $\sqrt{x}$?

Go back to what u is:

$u = 1+\sqrt{x} \Rightarrow$

$u-1 = \sqrt{x}$!

$$= \int_{x=0}^{x=1} \frac{2(u-1)\,du}{u^4}$$

$$= 2\int_{0=x}^{1=x} \left(\frac{u}{u^4} - \frac{1}{u^4}\right) du = 2\int_{x=0}^{x=1} (u^{-3} - u^{-4})\, du \text{, etc.}$$

$$= 2\left[\frac{u^{-2}}{-2} - \left(\frac{u^{-3}}{-3}\right)\right]_{x=0}^{x=1} = 2\left[\frac{(1+\sqrt{x})^{-2}}{-2} + \frac{(1+\sqrt{x})^{-3}}{3}\right]_0^1$$

$$= 2\left[\frac{(1+1)^{-2}}{-2} + \frac{(1+1)^{-3}}{3} - \left(\frac{1^{-2}}{-2} + \frac{1^{-3}}{3}\right)\right]$$

$$= 2\left[\frac{2^{-2}}{-2} + \frac{2^{-3}}{3} + \frac{1}{2} - \frac{1}{3}\right]$$

$$= 2\left[\frac{1}{-2\cdot 2^2} + \frac{1}{2^3 3} + \frac{3-2}{6}\right]$$

$$= 2\left[\frac{1}{-8} + \frac{1}{24} + \frac{1}{6}\right]$$

$$= 2\left[\frac{-3+1+4}{24}\right] = 2\left[\frac{2}{24}\right] = \boxed{\frac{1}{6}}$$

$$\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$$

$u = 1+\sqrt{x} = 1+x^{\frac{1}{2}}$

$\Rightarrow du = \frac{1}{2}x^{-\frac{1}{2}}dx \Rightarrow$

$dx = 2x^{\frac{1}{2}}du = 2\sqrt{x}\,du$

$$= \int_1^2 \frac{2\sqrt{x}\,du}{u^4}$$

$$\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$$

$$\boxed{u = 1+2x} \longrightarrow u-1 = 2x \longrightarrow \boxed{x = \frac{u-1}{2}}$$

$$du = 2dx \longrightarrow \boxed{dx = \frac{du}{2}}$$

$$u(0) = 1+2(0) = 1$$

$$u(4) = 1+2(4) = 9$$

$$= \int_1^9 \frac{x\ \frac{du}{2}}{\sqrt{u}} = \int_1^9 \frac{\left(\frac{u-1}{2}\right)\cdot\frac{du}{2}}{\sqrt{u}} =$$

$$\frac{1}{4}\int_1^9 \frac{u-1}{u^{1/2}}\,du = \frac{1}{4}\int_1^5 \left(\frac{u^1}{u^{1/2}} - \frac{1}{u^{1/2}}\right)du = \frac{1}{4}\int_1^5 \left(u^{1-\frac{1}{2}} - u^{-\frac{1}{2}}\right)du$$

Miscopied the "9!"

$$= \frac{1}{4}\int_1^5 \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du = \frac{1}{4}\left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]_1^5$$

$$= \frac{1}{4}\left[\frac{2}{3}(5)^{\frac{3}{2}} - 2(5)^{\frac{1}{2}} - \left(\frac{2}{3}(1)^{\frac{3}{2}} - 2(1)^{\frac{1}{2}}\right)\right]$$

$$= \frac{1}{4}\left[\frac{10}{3}(5)^{\frac{1}{2}} - 2(5)^{\frac{1}{2}} - \frac{2}{3} + 2\right] = \frac{1}{4}\left[\left(\frac{10}{3} - 2\right)\sqrt{5} + \frac{4}{3}\right]$$

$$= \cancel{\frac{1}{4}}\left[\frac{\cancel{4}}{3}\sqrt{5} + \frac{\cancel{4}}{3}\right] = \frac{\sqrt{5}+1}{3}$$

$$(5\cdot5\cdot5)^{\frac{1}{2}} = (5\cdot5)^{\frac{1}{2}}(5)^{\frac{1}{2}} = 5\cdot5^{\frac{1}{2}}$$

$$5^{\frac{3}{2}} = 5^{\frac{2+1}{2}} = 5^{\frac{2}{2}+\frac{1}{2}} = 5\cdot5^{\frac{1}{2}} = 5\sqrt{5}$$

$$\frac{1}{4}\int_1^9 \frac{u-1}{u^{1/2}}\,du = \frac{1}{4}\int_1^9\left(\frac{u^1}{u^{1/2}} - \frac{1}{u^{1/2}}\right)du = \frac{1}{4}\int_1^9\left(u^{1-\frac{1}{2}} - u^{-\frac{1}{2}}\right)du$$

$$= \frac{1}{4}\int_1^9\left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du = \frac{1}{4}\left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]_1^9$$

$$= \frac{1}{4}\left[\frac{2}{3}(9)^{\frac{3}{2}} - 2(9)^{\frac{1}{2}} - \left(\frac{2}{3}(1)^{\frac{3}{2}} - 2(1)^{\frac{1}{2}}\right)\right]$$

$$= \frac{1}{4}\left[\frac{2}{3}(27) - 2(3) - \left(\frac{2}{3} - 2\right)\right]$$

$$= \frac{1}{4}\left[18 - 6 + \frac{4}{3}\right] = \frac{1}{4}\left[12 + \frac{4}{3}\right] = \frac{1}{4}\left[\frac{40}{3}\right] = \frac{10}{3}$$

$$= 3.\overline{3}$$