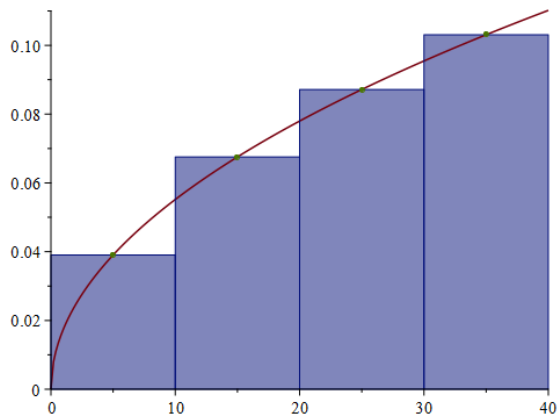
Giving Victor's question a better treatment. My intuition told me that degrees instead of radians was why

$$\int_0^{40} \sin(\sqrt{x})\,dx$$

Why'd my midpoint method give a positive when it should have been negative?

If you're in degrees mode, then $\sqrt{x}$ ranges from 0 to $\sqrt{40} \approx 6.$

So $\sin(\sqrt{x})$ goes up from $y=0$ to $y=\sin(2\sqrt{10}) \approx .1106$



Area $\approx \Delta x \sum_{k=1}^{4} f(x_k)$

$\frac{40}{4} = 10 = \Delta x$

Midpoints:

at $\frac{\Delta x}{2} = 5 = x_1$

$x_2 = 5 + \Delta x = 15$

$x_3 = 15 + \Delta x = 25$

$x_4 = 25 + \Delta x = 35$

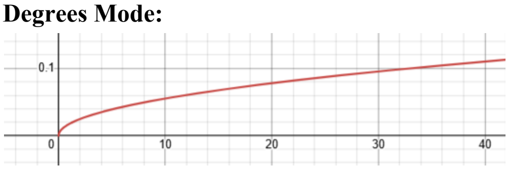
$$\Delta x \sum_{k=1}^{4} f(x_k) = 10 \sum_{k=1}^{4} \sin(\sqrt{x_k}) = 10\left[\sin(\sqrt{x_1}) + \sin(\sqrt{x_2}) + \sin(\sqrt{x_3}) + \sin(\sqrt{x_4})\right]$$

$]$

$$= 10\left[\sin(\sqrt{5}) + \sin(\sqrt{15}) + \sin(\sqrt{25}) + \sin(\sqrt{35})\right)$$

$\approx 10[$ $\qquad \frac{2k-1}{2}\Delta x = 5(2k-1) = 10k-5$

Degrees Mode:



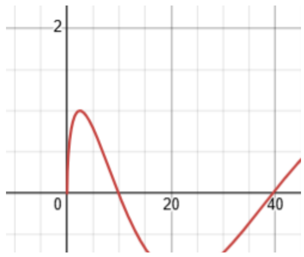
$f(x) = \sin(\sqrt{x})$

$10 \sum_{k=1}^{4} f(10k-5)$

Positive Area:

$= 2.96789121907$

Radians Mode:



$f(x) = \sin(\sqrt{x})$

$10 \sum_{k=1}^{4} f(10k-5)$

Negative Area:

$= -11.9899576333$

$$f(x) = \frac{x^2 + 4x - 32}{|x-4|} = \frac{(x-4)(x+8)}{|x-4|}$$

$$= \begin{cases} \dfrac{(x-4)(x+8)}{x-4} & \text{if } x \quad 4 \\ \dfrac{(x-4)(x+8)}{-(x-4)} & \text{if } x < 4 \end{cases}$$

$$= \begin{cases} x+8 & \text{if } x > 4 \quad \xrightarrow{x \to 4^+} \quad \boxed{12 = \lim_{x \to 4^+} f(x)} \quad \text{(b)} \\ -(x+8) & \text{if } x < 4 \quad \xrightarrow{x \to 4^-} \quad \boxed{-12 = \lim_{x \to 4^-} f(x)} \quad \text{(a)} \end{cases}$$

(c) $\boxed{\lim_{x \to 4^-} f(x) = -12 \neq 12 = \lim_{x \to 4^+} f(x) \longrightarrow \lim_{x \to 4} f(x) \nexists}$

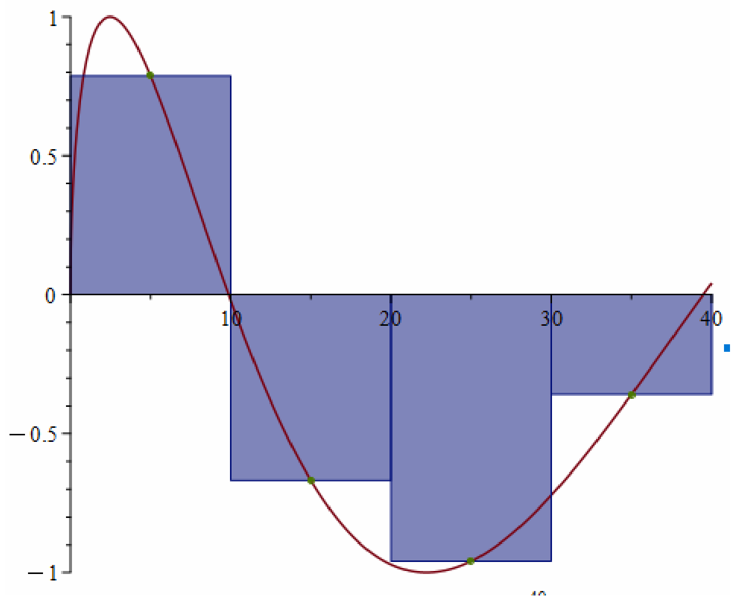
(a) $\lim_{x \to 4^-} \frac{x^2+4x-32}{|x-4|} = \lim_{x \to 4^-} \frac{(x+8)\cancel{(x-4)}}{-\cancel{(x-4)}} = \lim_{x \to 4^-} \frac{(x+8)}{-1} = -12$

(b) $\lim_{x \to 4^+} \frac{(x+8)(x-4)}{x-4} = \lim_{x \to 4^+} (x+8) = 12$

(c) $\lim_{x \to 4^-} f(x) = -12 \neq 12 = \lim_{x \to 4^+} f(x) \longrightarrow$

$\lim_{x \to 4} f(x) \nexists.$

4.2 #3



An animated Riemann sum midpoint approximation of $\int_0 f(x)\,dx$, where (x) = $\sin(\sqrt{x})$ and the partition is uniform. The approximate value of the tegral is -11.98995763. Number of subintervals used: 4.

$-12.54540695 \approx$ Actual (signed) area.

$$10\left(\sum_{i=0}^{3}\sin\left(\sqrt{10\,i+5}\right)\right)$$

$\frac{b-a}{n} = \frac{40-0}{4} = 10 = \Delta x$

$a + \frac{2(k-1)}{n}$

$x_1 = 0+5 \quad = a + \frac{\Delta x}{2}$

$x_2 = 15 = a + \frac{\Delta x}{2} + \Delta x = \frac{3\Delta x}{2} + a$

$x_3 = 25 = \frac{5\Delta x}{2} + a$

$x_4 = 35 = \frac{7\Delta x}{2} + a$

$k=1 \quad a + \frac{k}{2}\Delta x$

$a + \frac{3k}{2}\Delta x$

$\vdots$

$a + \frac{(2k-1)}{2}(\Delta x)$

$= a + (2k-1)(5)$

$= 0 \quad 10k-5$

$10\sum_{k=1}^{4}\sin\sqrt{10k-5}$

Maple did this

$10\sum_{k=0}^{3}\sin\sqrt{10k+5}$

$\left(\frac{2k+1}{2}\right)(10)$

$= (2k+1)(5)$

$= 10k+5$

#2 $f(x) = \begin{cases} x^2-5x-14 & \text{if } x<3 \\ 3x-29 & \text{if } x \geq 3 \end{cases}$

$x^2-5x-14 = \left(x^2-5x+\left(\frac{5}{2}\right)^2\right) - \frac{25}{4} - \frac{14(4)}{4}$

$= \left(x-\frac{5}{2}\right)^2 - \frac{81}{4} \overset{\text{SET}}{=} 0$

$\longrightarrow x = \frac{5 \pm 9}{2} \to 7, -2 \qquad (h,k) = \left(\frac{5}{2}, -\frac{81}{4}\right)$

$\lim_{x\to 3^-} f(x) = 3^2-5(3)-14 = 9-15-14 = -20$

$(3,-20)$ ○ "<3"

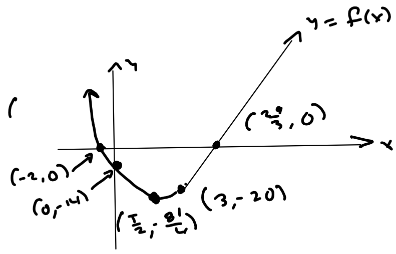
$3x-29=0$

$3x=29$

$x=\frac{29}{3} = 9.\overline{6}$

$\lim_{x\to 3^+} f(x) = 3(3)-29 = -20$

$(3,-20)$ ● "≥ 3"



Cnt's on $(-\infty,\infty)$, b/c $\lim_{x\to 3} f(x) = f(3)$ handles the only difficulty

Dif'bl on both pieces

@ $x=3$, we need to check that

$\lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$ $\exists$.

#2b Differentiability It's poly. on each piece, so difbl on $(-\infty,3)\cup(3,\infty)$ $\boxed{\text{Check } x=3}$

$f'(x) = \begin{cases} 2x-5 & \text{if } x<3 \\ 3 & \text{if } x>3 \end{cases}$

$f(x) = \begin{cases} x^2-5x-14 & \text{if } x<3 \\ 3x-29 & \text{if } x\geq 3 \end{cases} \longrightarrow$ what about @ $x=3$?

$\lim_{h\to 0^-} \frac{f(3+h)-f(3)}{h} = 2(3)-5 = 1$

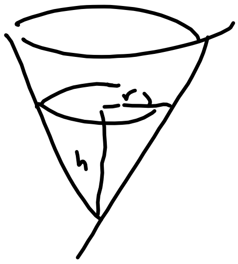
$\lim_{h\to 0^+} \frac{f(3+h)-f(3)}{h} = 3 \neq 1 = \lim_{h\to 0^-} \frac{f(3+h)-f(3)}{h} \Rightarrow$

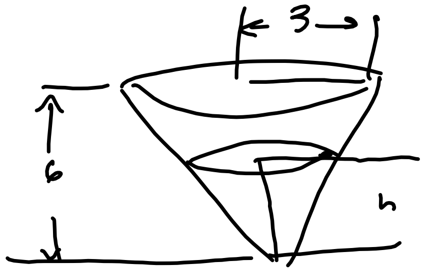
Not difbl @ $x=3$.

$f'_-(3) = 3$

$f'_+(3) = 1$

$\frac{dV}{dt} = -3\, m^3/min$





$\frac{r}{h} = \frac{3}{6}$ by similar triangles.

$\frac{r}{h} = \frac{1}{2}$ Want $\left.\frac{dh}{dt}\right|_{h=4}$

Lexicon

h = height (depth) of the water. (m)

r = radius of water surface (m)

t = time in minutes

$\frac{r}{h} = \frac{1}{2} \Rightarrow r = \frac{h}{2}$

V = volume (in m^3)

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{1}{12}\pi h^3$$

$$\Rightarrow \frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt} = 3\,\frac{m^3}{min} \cancel{= \frac{1}{4}\pi(4)^2\frac{dh}{dt}}$$

$$\Rightarrow \left.\frac{dV}{dt}\right|_{h=4} = \frac{1}{4}\pi(4^2)\left.\frac{dh}{dt}\right|_{h=4} = 3 \longrightarrow$$

m^2

$$(m^2)\, 4\pi \left.\frac{dh}{dt}\right|_{h=4} = 3\,\frac{m^3}{min} \Rightarrow$$

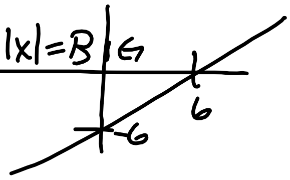
$$\left.\frac{dh}{dt}\right|_{h=4} \frac{3}{4\pi}\,\frac{\frac{m^3}{min}}{m^2} = \boxed{\frac{3}{4\pi}\,\frac{m}{min}}$$

$$|☺| = \begin{cases} ☺ & \text{if } ☺ \geq 0 \\ -☺ & \text{if } ☺ < 0 \end{cases}$$

$$|f| = \begin{cases} f & \text{if } f \geq 0 \\ -f & \text{if } f < 0 \end{cases}$$

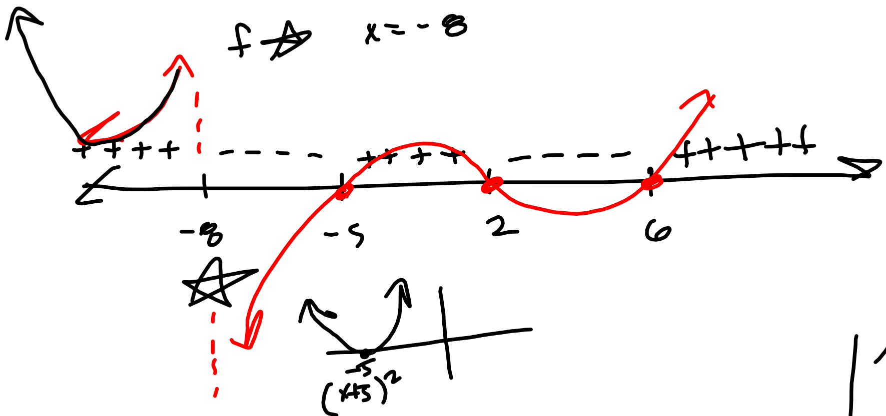
Your job:
sign pattern for f.

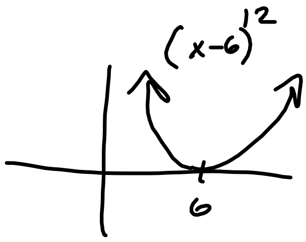
$$f(x) = \frac{(x-2)(x+5)(x-6)^1}{(x+8)} \approx \frac{x^3}{x} \text{ for } |x| = B \mid 6$$



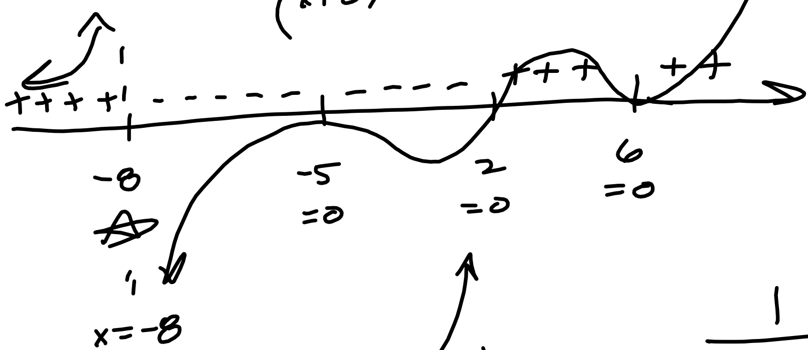
$f = 0: \quad x = 2, -5, 6$

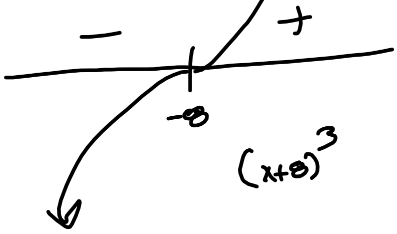
$f \not\exists \quad x = -8$



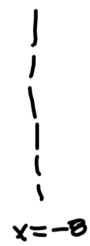


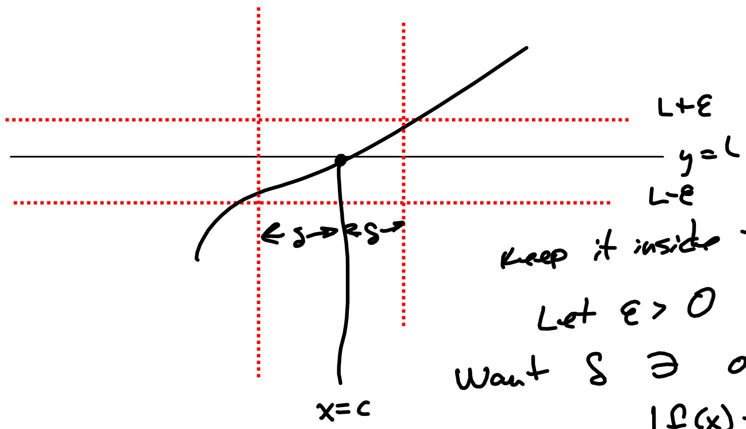
$$f(x) = \frac{(x-2)^1 (x+5)^2 (x-6)^{12}}{(x+8)^3}$$





$$\frac{1}{(x+8)^3}$$





Keep it inside the ε-tube

Let $\varepsilon > 0$

Want $\delta \ni 0 < |x-c| < \delta$

to give $|f(x) - L| < \varepsilon$

$\lim_{x \to c} f(x) = L$ means, given $\varepsilon > 0$, I can find

$\delta > 0 \ni 0 < |x-c| < \delta \Rightarrow |f(x) - L| < \varepsilon$

$\lim_{x \to -3}$

$\lim_{x \to -2} 5x+7 = -3$ $L=-3$ $f(x) = 5x+7$

$c=-2$

Let $\varepsilon > 0$ be given.

want $\delta > 0 \ni 0 < |x-c| < \delta \Rightarrow$

WANT $|f(x)-L| < \varepsilon \Leftarrow$

$|5x+7-(-3)|$

You did $|5x+7-3| = |5x+4| = 5|x+\frac{4}{5}|$

~~st~~ $|5x+7-(-3)| = |5x+10| = 5|x+2|$

$= 5|x-(-2)| \overset{\text{WANT}}{<} \varepsilon$

$\Rightarrow |x-(-2)| < \frac{\varepsilon}{5}$

Define $\delta = \frac{\varepsilon}{5}$

Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{5}$. Then

$0 < |x-c| = |x-(-2)| < \delta \Rightarrow$

$|f(x)-L| = |5x+7-(-3)| = |5x+10| = 5|x+2|$

$= 5|x-(-2)| < 5\delta = 5 \cdot \frac{\varepsilon}{5} = \varepsilon$ ∎

$$\lim_{x \to 5} (3x-7) = 8$$

WANT $|3x-7-8| = |3x-15| < \varepsilon$

$\Rightarrow 3|x-5| < \varepsilon$

$\Rightarrow \underbrace{|x-5|}_{<\delta} < \frac{\varepsilon}{3} \equiv \delta$

Proof $0 < |x-5| < \delta \Rightarrow$

$|f(x) - L| = |3x-7-8| = 3|x-5| < 3\delta = 3 \cdot \frac{\varepsilon}{3} = \varepsilon$