**Theory for 4.3 (1st and 2nd Fundamental Theorems of Calculus)**

https://harryzaims.com/public_html/201/videos/chapter-04/4-3/4-3-notes.pdf

$f(x) = 2x^2 + 9$ Find estimate

Right Endpoints

$x_k = a + k\frac{b-a}{n}$

Left :

$x_k = a + (k-1)\frac{b-a}{n}$

$$\frac{k\left(\frac{b-a}{n}\right) + (k-1)\left(\frac{b-a}{n}\right)}{2}$$

$$\frac{(k + k-1)\left(\frac{b-a}{n}\right)}{2}$$

$$= \frac{(2k-1)\left(\frac{b-a}{n}\right)}{2}$$

Mid point:

$a + \frac{(2k-1)}{2}\left(\frac{b-a}{n}\right)$

$\boxed{\text{3 RECT}}$ $a + k\left(\frac{b-a}{n}\right) = -1 + k\left(\frac{2-(-1)}{3}\right) = -1 + k$

Area $\approx 1\sum_{k=1}^{3} f(-1+k) =$ 37

$\boxed{\text{6 Rect}}$ $\frac{b-a}{n} = \frac{3}{6} = \frac{1}{2}$

$x_k = -1 + k\left(\frac{1}{2}\right) = -1 + \frac{k}{2}$

$\Delta x = \frac{b-a}{n} = \frac{1}{2}$

$\Delta x \sum_{k=1}^{n} f(x_k) = \frac{1}{2}\sum_{k=1}^{6} f\left(-1 + \frac{k}{2}\right) =$ 34.75

$f(x) = 2x^2 + 9$

$1 \cdot \sum_{k=1}^{3} f(-1+k)$

$= 37$

$\frac{1}{2} \cdot \sum_{k=1}^{6} f\left(-1+\frac{k}{2}\right)$

$= 34.75$

In general, (Right) Area $\approx \frac{b-a}{n} \sum_{k=1}^{n} f\left(a + \frac{b-a}{n} k\right)$

$= \frac{3}{n} \sum_{k=1}^{n} f\left(a + \frac{3}{n} k\right)$

$\frac{3}{100} \cdot \sum_{k=1}^{100} f\left(-1+\frac{3k}{100}\right)$

$= 33.0909$

$\frac{3}{1000} \cdot \sum_{k=1}^{1000} f\left(-1+\frac{3k}{1000}\right)$

$= 33.009009$

$\frac{3}{10000} \cdot \sum_{k=1}^{10000} f\left(-1+\frac{3k}{10000}\right)$

$= 33.00090009$

$$\sum_{k=1}^{n} 5 = 5n$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} = \frac{n^2+n}{2} = \frac{n^2 + \text{lower degree}}{2} = \frac{n^2 + \cdots}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3 + \cdots}{6} = \frac{n^3 + \cdots}{3}$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{n^2+n}{2}\right)^2 = \frac{n^4 + \cdots}{4}$$

$$\int_1^5 x^2\,dx = \lim_{n\to\infty} \frac{5-1}{n} \sum_{k=1}^{n} f(x_k) =$$

$$\lim_{n\to\infty} \frac{5-1}{n} \sum_{k=1}^{n} x_k^2 = \lim_{n\to\infty} \frac{5-1}{n} \sum_{k=1}^{n} \left(\frac{4k}{n}+1\right)^2 =$$

$=$

$$x_k = a + k\Delta x$$
$$= 1 + k\left(\frac{4}{n}\right)$$
$$= \frac{4k}{n} + 1$$

So, in practice:

① Find $\Delta x = \frac{b-a}{n}$ ② $x_k = a + k\Delta x$

③ Plug in to $f$ & write the sum

④ Pass to the limit

$$\frac{4}{n}\sum_{k=1}^{n}\left(\frac{4k}{n}+1\right)^2 = \frac{4}{n}\sum_{k=1}^{n}\left(\frac{16k^2}{n^2}+\frac{8k}{n}+1\right)$$

$$= \frac{4}{n}\left[\sum_{k=1}^{n}\frac{16k^2}{n^2} + \sum_{k=1}^{n}\frac{8k}{n} + \sum_{k=1}^{n}1\right]$$

$$= \frac{4}{n}\cdot\frac{16}{n^2}\sum_{k=1}^{n}k^2 + \frac{4}{n}\cdot\frac{8}{n}\sum_{k=1}^{n}k + \frac{4}{n}\cdot n$$

$$= \frac{64}{n^3}\left[\frac{n^3+\cdots}{3}\right] + \frac{32}{n^2}\left[\frac{n^2+\cdots}{2}\right] + 4$$

$$\xrightarrow{n\to\infty} \frac{64}{3}+\frac{32}{2}+4$$

$$= \frac{128+96+24}{6} = \frac{248}{6} = \boxed{\frac{124}{3}} =$$

$$\begin{array}{r} 152 \\ 96 \\ \hline 248 \end{array}$$

$$\int_1^5 x^2\,dx = 41.3333333333$$

$$\frac{124}{3} = 41.3333333333$$

**In Desmos, type "int" and it'll spit out the integral for you to edit!!!!**

#4 §4.1

$n = 10, 30, 50, 100$

$f(x) = 2\cos(x)$ on $[0, \frac{\pi}{2}]$

$\Delta x = \frac{b-a}{n} = \frac{\frac{\pi}{2}}{n} = \frac{\pi}{2n}$

$x_k = 0 + k\left(\frac{\pi}{2n}\right) = \frac{\pi k}{2n}$

$n=10$:

$\Delta x = \frac{\pi}{20}$

$x_k = \frac{\pi k}{20}$

0.919403170015

$n=30$ : $\Delta x = \frac{\pi}{60}$

$x_k = 0 + \frac{\pi k}{2n} = \frac{\pi k}{60}$

$g(x) = 2\cos(x)$

$$\frac{\pi}{20}\sum_{k=1}^{10}\cos\left(\frac{\pi k}{20}\right)$$

$= 0.919403170015$

$$\frac{\pi}{60}\sum_{k=1}^{30}\cos\left(\frac{\pi k}{60}\right)$$

$= 0.973591587715$

**Net Change:** Rate function integrand

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$ is Net Change in $F$ over $[a,b]$.

Total Change

$$\int_a^b |F'(x)|\,dx$$

**See the Table of Antiderivatives in 3.9, 4.3, etc.**

§ 4.2 painful

§ 4.1 "

§ 4.3 Finally!

$\boxed{\text{FTC I}}$

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

$\boxed{\text{FTC I Chain Rule}}$

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f(g(x))\,g'(x)$$

$$h(x) = \int_0^x \sin(t)\,dt \Rightarrow$$

$$h'(x) = \sin(x)$$

$$h(x) = \int_0^{x^2-5x} \sin(t)\,dt$$

$$h'(x) = \left(\sin(x^2-5x)\right)(2x-5)$$

FTC II :

If $F'(x) = f(x)$, then

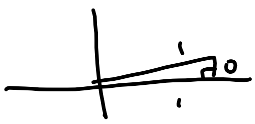
$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$\forall$ $F$ satisfying $F'(x) = f(x)$

$$\int_0^{\frac{3\pi}{4}} \sin(x)\,dx = -\cos(x)\Big]_0^{\frac{3\pi}{4}} = -\cos\left(\tfrac{3\pi}{4}\right) - \left(-\cos(0)\right)$$


$$= -\left(-\tfrac{1}{\sqrt{2}}\right) - (-1)$$

$$= \boxed{\tfrac{1}{\sqrt{2}} + 1}$$




$$\int_0^{\frac{3\pi}{4}} \sin(x)\,dx$$

$= 1.70710678119$

$$\frac{1}{\sqrt{2}} + 1$$

$= 1.70710678119$