4.1 #1 - Book Answers:

a. **23.2**

b. **16.1 and 20.7**

3 rectangles, Right Endpoints Victor's version

4.1 #2 over x = -1 to x = 2

3 + 4x^2 for Victor

8 + 4x^2 for Mark

$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{3} = \frac{3}{3} = 1$

$x_k = a + k\Delta x = -1 + k(1) = -1 + k$

$x_1 = a + 1 = -1 + 1 = 0$

$x_2 = a + 2(1) = -1 + 2 = 1$

$x_3 = a + 3(1) = -1 + 3 = 2$

$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{3} f(-1+k)\Delta x = \frac{b-a}{3}\sum_{k=1}^{3}(3 + 4x_k^2)$

$= \frac{3}{3}\sum_{k=1}^{3} 3 + 4(-1+k)^2$ $\quad (1+k)$

$= (3 + 4(0)^2) + 3 + 4(1)^2 + 3 + 4(2)^2$

$= 3 + (3+4) + (3+16) = 3 + 7 + 19 = 29$

$$\sum_{k=1}^{n} k = 1+2+3+4+\cdots+n = \frac{n(n+1)}{2} = \frac{n^2+n}{2} = \frac{n^2+\sim}{2}$$, where

"$\sim$" is all the lower-degree stuff.

Sequel:

$$\lim_{n\to\infty} \frac{7}{n^2}\left[\frac{n^2+\sim}{2}\right] = 7\left(\frac{1}{2}\right) = \frac{7}{2}$$

$$\frac{7}{n^2}\left[\frac{n^2}{2}+\frac{\sim}{2}\right] = \frac{7n^2}{n^2} + \frac{7\sim}{2n^2} \to 0$$

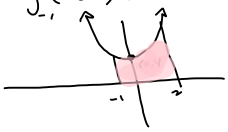
Important

$$\sum_{k=1}^{n} k^2 = 1^2+2^2+\cdots+n^2$$

$$= \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+\sim}{6} = \frac{n^3+\sim}{3}$$

$$\sum_{k=1}^{n} k^3 = 1^3+2^3+\cdots+n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{(n^2+\sim)^2}{2^2} = \frac{n^4+\sim}{4}$$

$$\int_{-1}^{2}(4+8x^2)\,dx = \text{Area under } 4+8x^2 \text{ between } -1 \text{ \& } 2$$



$$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$$

$$x_k = a+k\Delta x = -1+k\left(\frac{3}{n}\right)$$

$$f(x_k) = 4+8x_k^2 = 4+8\left(-1+\frac{3k}{n}\right)^2$$

$$= 4+8\left(\frac{9k^2}{n^2}-\frac{6k}{n}+1\right)$$

$$= 12+\frac{72k^2}{n^2}-\frac{48k}{n}$$

$$\left(-1+\frac{3k}{n}\right)^2 = \left(\frac{3k}{n}-1\right)^2$$

$$= \frac{9k^2}{n^2}-\frac{2\cdot 3k}{n}+1$$

$$= \frac{9k^2}{n^2}-\frac{6k}{n}+1$$

$$= x_k^2$$

Put it together:

$$\frac{b-a}{n}\sum_{k=1}^{n} f(x_k) = \frac{3}{n}\sum_{k=1}^{n}\left(\frac{72k^2}{n^2}-\frac{48k}{n}+12\right)$$

$$= \frac{3}{n}\left[\sum_{k=1}^{n}\frac{72k^2}{n^2} - \sum_{k=1}^{n}\frac{48k}{n} + \sum_{k=1}^{n}12\right]$$

$$= \frac{3}{n}\left[\frac{72}{n^2}\sum_{k=1}^{n}k^2 - \frac{48}{n}\sum_{k=1}^{n}k + 12\sum_{k=1}^{n}1\right]$$

$$= \frac{72(3)}{n^3}\left(\frac{n^3+\sim}{3}\right) - \frac{3(48)}{n^2}\left(\frac{n^2+\sim}{2}\right) + \frac{36}{n}\cdot n$$

$$\xrightarrow{n\to\infty} \frac{3(72)n^3}{3n^3} - \frac{3(\cancel{48}^{24})n^2}{\cancel{2}n^2} + 36$$

?!

$$= 72-72+36 = 36 = \int_{-1}^{2}(4+8x^2)\,dx$$

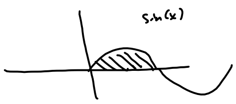
2nd Fundamental Theorem of Calculus:

$$\int_{-1}^{2}(4+8x^2)\,dx = 4x+\frac{8}{3}x^3\Big]_{-1}^{2} = 4(2)+\frac{8}{3}(2)^3-\left(4(-1)+\frac{8}{3}(-1)^3\right)$$

$$= 8+\frac{8}{3}(8)-\left(-4-\frac{8}{3}\right)$$

$$= 8+\frac{64}{3}+4+\frac{8}{3}$$

$$= 12+\frac{72}{3} = 12+24 = 36$$



Area under sine from 0 to π

$$\int_0^{\pi}\sin(x)\,dx = -\cos(x)\Big]_0^{\pi} = -\cos(\pi)-(-\cos(0))$$

$$= -(-1)-(-1)$$

$$= 1+1 = 2$$

$$\int_0^{2\pi}\sin(x)\,dx = -\cos(x)\Big]_0^{2\pi} = -\cos(2\pi)-(-\cos(0))$$

$$= -1+1 = 0.$$

These represent SIGNED AREAS.

$\int_a^b f(x)\,dx =$ Net Change.

$\int_a^b |f(x)|\,dx =$ Total Change

Left-endpoint

$$x_k = a + (k-1)\Delta x$$

$$= a + (k-1)\frac{b-a}{n}$$

Midpoint

$$x_1 = a + \frac{1}{2}\Delta x$$

$$x_2 = a + \frac{1}{2}\Delta x + \Delta x$$

$$= a + \frac{3}{2}\Delta x$$

$$x_3 = a + \frac{5}{2}\Delta x$$

$x_k = a + \frac{2k-1}{2}\Delta x$ Midpoint inputs x_k.

Using Desmos for these finite, but messy sums

$f(x) = 4 + 8x^2$

$$\frac{1}{2}\sum_{n=1}^{6} f\left(-1 + n\left(\frac{1}{2}\right)\right) = 43$$

$n=6$

$$\frac{(2-(-1))}{10} = 0.3$$

$\frac{b-a}{n} = \frac{2-(-1)}{n} = \frac{3}{n}$

$$\frac{3}{10}\sum_{n=1}^{10} f\left(-1 + n\left(\frac{3}{10}\right)\right) = 39.96$$

$n=10$ $\frac{b-a}{n}\sum_{k=1}^{n} f\left(a + k\left(\frac{b-a}{n}\right)\right)$

$$\frac{3}{50}\sum_{n=1}^{50} f\left(-1 + n\left(\frac{3}{50}\right)\right) = 36.7344$$

$n=50$

$$\frac{3}{100}\sum_{n=1}^{100} f\left(-1 + n\left(\frac{3}{100}\right)\right) = 36.3636$$

$n=100$

$$\frac{3}{1000}\sum_{n=1}^{1000} f\left(-1 + n\left(\frac{3}{1000}\right)\right) = 36.036036$$

$n=1000$

$$\frac{3}{10000}\sum_{n=1}^{10000} f\left(-1 + n\left(\frac{3}{10000}\right)\right) = 36.00360036$$

$n=10000$
Looks like it's approach $A=36$.