Antiderivatives Assume $C \in \mathbb{R}$ & constant

$\int x\,dx = \frac{x^2}{2} + C$ → $\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad \forall\, n \neq -1}$ Power Rule

2 tracks of Discussion → The case $n=-1$ is very special

Trigonometric Anti-Derivatives

$\int \sin(x)\,dx = -\cos(x) + C$

$\int \cos(x)\,dx = \sin(x) + C$

$\int \tan(x)\,dx =$ No idea

$\int \cot(x)\,dx =$ No idea

$\int \sec(x)\,dx =$ " "

$\int \csc(x)\,dx =$ " "

Using logarithms

$\int \sec^2(x)\,dx = \tan(x) + C$

$\int \csc^2(x)\,dx = -\cot(x) + C$

$\int \sec(x)\tan(x)\,dx = \sec(x) + C$

$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$

SNEAK PREVIEW

$\int x^{-1}\,dx = \underbrace{\int \frac{1}{x}\,dx = \ln|x| + C}$

This and the chain rule will give us

The foundation of it all

$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$

$$\frac{d}{dx}\left[\ln(f(x))\right] = \frac{1}{f(x)} \cdot f'(x) = \frac{f'(x)}{f(x)}.$$

$$\int \tan(x)\,dx = \int \frac{\sin(x)\,dx}{\cos(x)} = -\int \frac{\frac{d}{dx}(\cos(x))}{\cos(x)}\,dx$$

$$= -\ln(\cos(x))\ !\ + C$$

$$= \ln\left((\cos(x))^{-1}\right) + C$$

$$= \ln(\sec(x)) + C$$

Punchline:

$\int \tan(x)\,dx = \ln|\sec(x)| + C$

Week 9 #9

$f''(x) = x^{\frac{1}{3}} - \cos(x)$

$f'(x) = \frac{3}{4}x^{\frac{4}{3}} - \sin(x) + C \longrightarrow$

$f(x) = \frac{3}{4}\cdot\frac{3}{7}x^{\frac{7}{3}} + \cos(x) + Cx + D$

$f(0) = f(1) = 2$

$f(0) = \frac{9}{28}(0) + \cos(0) + C(0) + D = 2$

$\Longrightarrow D = 2 - \cos(0) = 2 - 1 = \boxed{1 = D}$

$f(1) = \frac{9}{28}(1) + \cos(1) + C + 1 = 2$

$C = 2 - \frac{9}{28} - 1 - \cos(1)$

$C = \frac{28-9}{28} - \cos(1)$

$= \boxed{\frac{19}{28} - \cos(1) = C}$

$\boxed{f(x) = \frac{9}{28}x^{\frac{7}{3}} + \cos(x) + Cx + D}$