§ 3.7

Sum is 23. Maximize the product

Let $x = 1^{st}$ #

$y = 2^{nd}$ #

Then $x + y = 23 \Longrightarrow y = 23 - x$

Maximize the product $xy = x(23-x) = 23x - x^2 = f(x)$

$\begin{array}{r} 23 \\ \times 23 \\ \hline 69 \\ 46 \\ \hline 529 \end{array}$

$\Longrightarrow f'(x) = 23 - 2x \overset{set}{=} 0 \Longrightarrow 23 = 2x \Rightarrow x = \frac{23}{2}$

$\frac{23}{2}\left(23 - \frac{23}{2}\right) = \left(\frac{23}{2}\right)^2 = \frac{523}{4} = 130.75$

when $x = y = \frac{23}{2}$

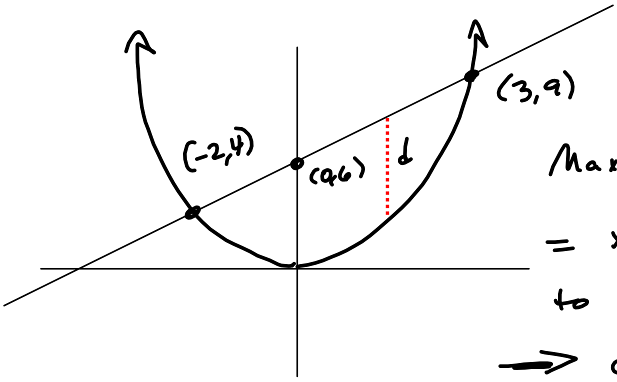
§ 3.7 #2

Max vertical distance between $y_1 = x+6$ & $y_2 = x^2$

for $x \in [-2,3]$

$y_1(-2) = 4 \qquad y_1(3) = 9$

$y_2(-2) = 4 \qquad y_2(3) = 9$



Maximize $d = y_1 - y_2$

$= x+6-(x^2) = -x^2+x+6 = d(x)$

to be maximized

$\Rightarrow d'(x) = -2x+1 \overset{\text{set}}{=} 0$

$-2x = -1$

$x = \frac{1}{2}$ &

$d(\frac{1}{2}) = -(\frac{1}{2})^2 + (-\frac{1}{2}) + 6$ $+\frac{1}{2}$

$= -\frac{1}{4} - \frac{2}{4} + \frac{24}{4} = \frac{21}{4}$

$+\frac{2}{4}$ $\quad \frac{25}{4}.$

Find the point on the line y = 3x + 4 that is closest to the origin. → Minimize the distance

Fact: the square root is an increasing function. That means to maximize sqrt("something"), just maximize "something."

Let (x,y) on graph of $f(x) = 3x+4$.

$\Rightarrow (x,y) = (x, 3x+4)$

Origin: $(x,y) = (0,0)$

Distance from $(x, 3x+4)$ to $(0,0)$ is

$$d = \sqrt{(x-0)^2 + (3x+4-0)^2}$$

$$= \sqrt{x^2 + 9x^2 + 24x + 16}$$

$$= \sqrt{10x^2 + 24x + 16}$$ To maximize d, we maximize

$g(x) = d^2(x) = 10x^2 + 24x + 16$ or

$\Rightarrow g'(x) = 20x + 24 \overset{SET}{=} 0 \Rightarrow$

$20x = -24$

$x = \frac{-24}{20} = \boxed{-\frac{6}{5} = x}$

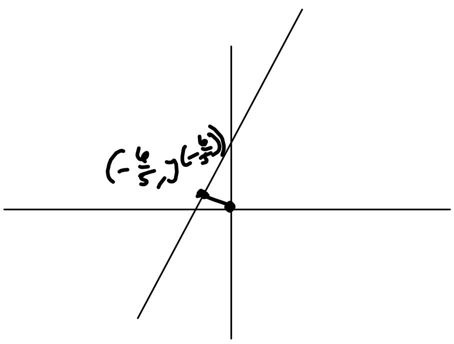
$$\begin{array}{r} {}^{2}24 \\ \times\ 6 \\ \hline 144 \end{array}$$

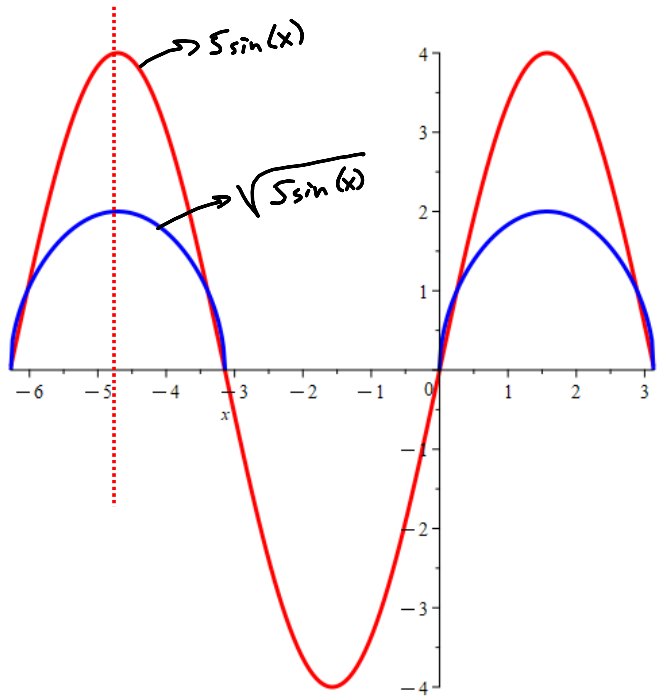
Now, $d\left(-\frac{6}{5}\right) = \sqrt{10\left(-\frac{6}{5}\right)^2 + 24\left(-\frac{6}{5}\right) + 16}$

$$= \sqrt{10\left(\frac{36}{25}\right) - \frac{144}{5} + \frac{16}{1}\cdot\frac{5}{5}}$$

$$= \sqrt{\frac{72 - 144 + 80}{5}} = \sqrt{\frac{8}{5}} = d.$$

$d = \frac{2\sqrt{2}}{\sqrt{5}} = \frac{2\sqrt{10}}{5}$ is minimum distance, found @ $x = -\frac{6}{5}$





They're both maximized at the same input value.

#8 on Week 9 Assignment is Newton's method

$f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

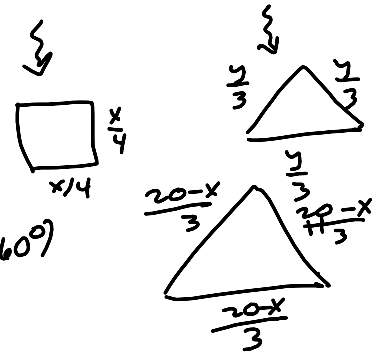
A piece of wire 20 m ong is cut into 2 pieces. One piece is bent into a square and the other is bent into an equilateral triangle.

a. How much wire should be used for the square in order to maximize the total area?

b. How much wire for the square if you want to minimize the area?

Let x = length of wire used for the square in m.
& y = " " " " " " triangle " ".

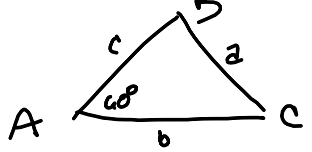
(a) we want to maximize the area

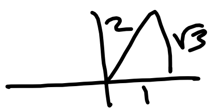


Area of triangle ABC:

$\text{Area} = \frac{1}{2} bc \sin A = \frac{1}{2}\left(\frac{20-x}{3}\right)\left(\frac{20-x}{3}\right)\sin(60^\circ)$

$= \frac{\sqrt{3}}{36}(20-x)^2$





Page of Bad Stuff (Mistakes)

$$\text{Area} = \left(\frac{x}{4}\right)^2 + \frac{\sqrt{3}}{36}(20-x)^2$$

$$\Rightarrow \frac{d\text{Area}}{dx} = 2\left(\frac{x}{4}\right)\left(\frac{1}{4}\right) + \frac{2\sqrt{3}}{36}(20-x)(-1)$$

$$= \frac{1}{8}x + \left(\frac{\sqrt{3}}{18}(20) + \frac{\sqrt{3}}{18}(-x)\right)(-1)$$

$$= \frac{1}{8}x - \frac{\sqrt{3}}{9} + \frac{\sqrt{3}}{18}x \overset{\text{SET}}{=} 0$$

Fixed to patch it up.

$$\frac{1}{8}x + \frac{\sqrt{3}}{18}x = \frac{\sqrt{3}}{9}$$

$$x\left(\frac{1}{8} + \frac{\sqrt{3}}{18}\right) = \frac{\sqrt{3}}{9}$$

$$x = \frac{\sqrt{3}}{9}\left(\frac{1}{\frac{1}{8} + \frac{\sqrt{3}}{18}}\right)$$

See Next Page(s)
I fixed my buffoonery after class.

A piece of wire 20 m ong is cut into 2 pieces. One piece is bent into a square and the other is bent into an equilateral triangle.

a. How much wire should be used for the square in order to maximize the total area?

b. How much wire for the square if you want to minimize the area?

> **This work spirals towards a solution, but it's not that well-written. If I were a student who took 3 pages to finally work it out, I would re-write the thing to be turned in if it were a written assignment. People tend to try to make their first draft "perfect" and it's better to just be messy and fast, and then put it all together in a nice little report.**

Let $x=$ length of wire used for the square in m.

& $y=$ " " " " " " triangle " " .

$\Rightarrow y = 20 - x$

$\Rightarrow$ Area of square is $\left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$

& Area of triangle $= \frac{1}{2}\left(\frac{20-x}{3}\right)\left(\frac{20-x}{3}\right)\sin 60^\circ$

$= \frac{1}{2}\left(\frac{20-x}{3}\right)^2\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{36}(20-x)^2$

$A=$ Area as a function of x is

$A(x) = \frac{x^2}{16} + \frac{\sqrt{3}}{36}(20-x)^2 \Rightarrow$

$A'(x) = \frac{x}{8} + 2\left(\frac{\sqrt{3}}{36}\right)(20-x)(-1)$

$= \frac{x}{8} - \frac{\sqrt{3}}{18}(20-x)$

$= \frac{x}{8} - \frac{20\sqrt{3}}{18} + \frac{\sqrt{3}}{18}x$ LCD $= 2\cdot2\cdot2\cdot3\cdot3$

$\Rightarrow \frac{x}{8} + \frac{\sqrt{3}}{18}x = \frac{10\sqrt{3}}{9}$

$$\frac{9x+4\sqrt{3}x}{72}=\frac{10\sqrt{3}}{9} \longrightarrow$$

$$81x+36\sqrt{3}x=720\sqrt{3}$$

$$x(81+36\sqrt{3})=720\sqrt{3}$$

$$x=\frac{720\sqrt{3}}{81+36\sqrt{3}} \approx \boxed{8.699290352 \approx x}$$

This is the answer to part b.

MINIMUM

$$\longrightarrow 20-x=\boxed{y \approx 11.30070965}$$

(b) Minimize it!

What's the Domain? $x \in [0,20]$

This belongs near the top!

Check end points

$$A(x)=\frac{x^2}{16}+\frac{\sqrt{3}}{36}(20-x)^2 \longrightarrow$$

$$A(0)=\frac{\sqrt{3}}{36}(20)^2=\frac{100\sqrt{3}}{9}\approx 19.24500898 \quad \text{Neither.}$$

$$A(20)=\frac{20^2}{16}=\boxed{25 = MAX} \text{ when } \boxed{x=20, y=0}$$

This is part a answer.

Min of 10.87411294 @ $x \approx$

$$A(8.699290352)\approx \boxed{10.87411294 \ m^2}$$

Min.

This is part b answer

I wasn't paying that close attention to the FACT that the Area was a quadratic function with a POSITIVE leading coefficient.

Of course the derivative gave us a local MINIMUM.

Then it was just a matter of checking the endpoints for which was bigger.