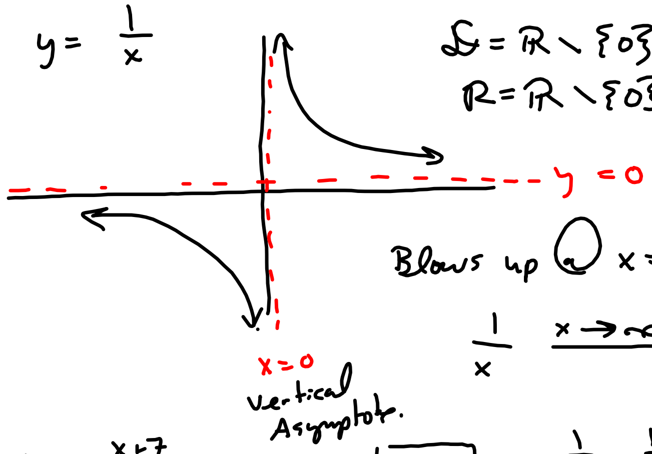
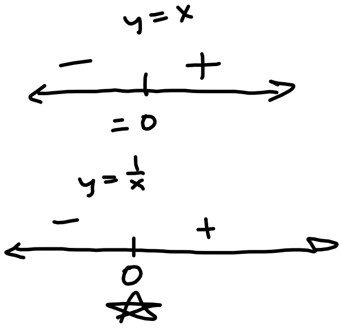
Rational Functions, End Behavior, Vertical Asymptotes

$y = \frac{1}{x}$



$\mathcal{D} = \mathbb{R} \setminus \{0\}$

$\mathcal{R} = \mathbb{R} \setminus \{0\}$



Blows up @ $x = 0$

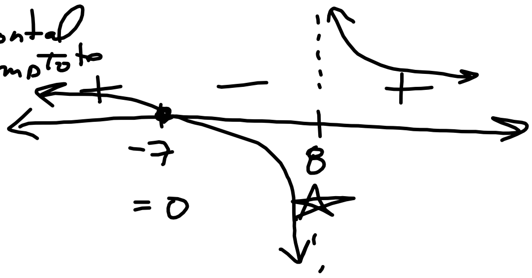
$\frac{1}{x} \xrightarrow{x \to \infty} 0$

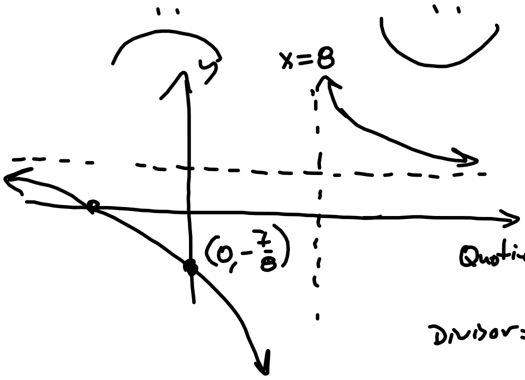
$\frac{1}{2}, \frac{1}{4}, \frac{1}{12}, \frac{1}{\text{Bezillion}} \approx 0$

$R(x) = \frac{x+7}{x-8} \xrightarrow{x \to \infty} \frac{x}{x} = \boxed{1 = y}$ Horizontal Asymptote

$x = 8$ V.A.

$(-7, 0) = x\text{-int}$





What's calculus say?

Quotient

$$\begin{array}{r} 1 \;\; r\; 15 = \text{remainder} \\ \text{Divisor} = x-8 \,\big)\, \overline{x+7} = \text{Dividend} \\ \underline{-(x-8)} \\ 15 \end{array}$$

This says

$R(x) = \frac{x+7}{x-8} = 1 + \frac{15}{x-8} = 1 + 15(x-8)^{-1} \longrightarrow R'(x) = -15(x-8)^{-2}$

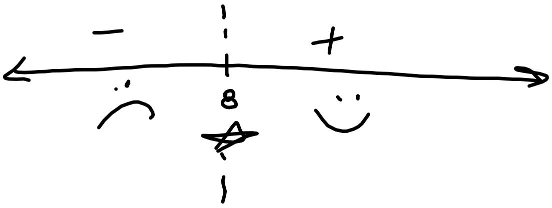
$= \frac{-15}{(x-8)^2}$

$x+7 = (x-8)(1) + 15$

$R(x) = \frac{x+7}{x-8} = \frac{u}{v} \longrightarrow f'(x) = \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

$= \frac{1(x-8) - (x+7)(1)}{(x-8)^2} = \frac{x-8-x-7}{(x-8)^2} = \frac{-15}{(x-8)^2} = R'(x)$

$R''(x) = \frac{d}{dx}\left[-15(x-8)^{-2}\right] = 15 \xrightarrow{} 30\; (x-8)^{-3} = \frac{15 \to 30}{(x-8)^3}$



Click on Link:

Rational Functions for College Algebra Students

This should really help review for rational functions for THIS class. Scroll down to #s 3, 4, and 5.

This class does some of the best work of any Calc I group I've had.

3.3, then rational functions, applications,
Newton's Method and Antiderivatives

It's all about zeros upstairs (= 0) & downstairs (☆)

$$R(x) = \frac{3x^2+6x-24}{4x^2+27x+18} = \frac{(3x-6)(x+4)}{(x+6)(4x+3)} = \frac{3(x-2)(x+4)}{(x+6)(4x+3)}$$

Nothing cancels so No Holes

$-6, -4, -\frac{3}{4}, 2$

$\mathcal{D} = \mathbb{R} \setminus \{-6, -\frac{3}{4}\}$ $\boxed{x=-6,\ x=-\frac{3}{4} \text{ are vertical asymptotes}}$

$R = 0 \Rightarrow x = -4, 2 \rightsquigarrow$ $\boxed{(-4,0),\ (2,0) \text{ are x-ints}}$

Limit @ infinity (END BEHAVIOR)

my way: $\frac{3x^2+\sim}{4x^2+\sim} \xrightarrow{|x|\to\infty} \frac{3x^2}{4x^2} = \boxed{\frac{3}{4} = y} = HA$. Nice when

deg(numer) = deg(denom)

$y = \frac{3}{4}$ = Horizontal Asymptote = ~~Hor. As.~~ = HA!

deg(numer) > deg(denom) Oblique Asymptotes ($ax+b$, ax^2+bx+c

$\frac{6x^2-5x-7}{x+2}$ $2-1=1$ degree $y = ax+b$

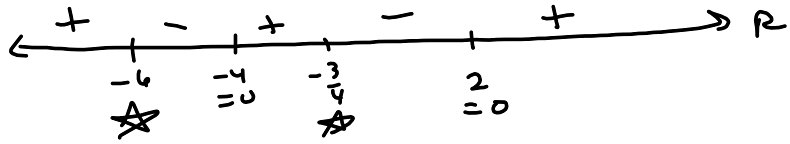
$\frac{6x^3+5x^2-7}{x+2}$ $3-1=2$ degree $y = ax^2+bx+c$

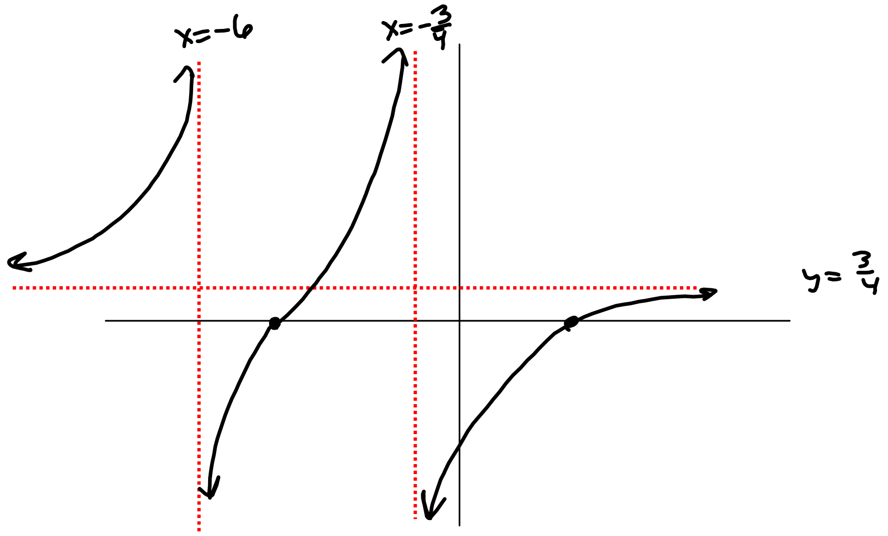
deg(numer) < deg(denom) ("Proper")

$\frac{7x}{x^2+2} \xrightarrow{x\to\pm\infty} 0$

$\frac{9x^{50}+7x^{40}-5x^{17}}{x^{51}+5} \xrightarrow{x\to\pm\infty} 0$

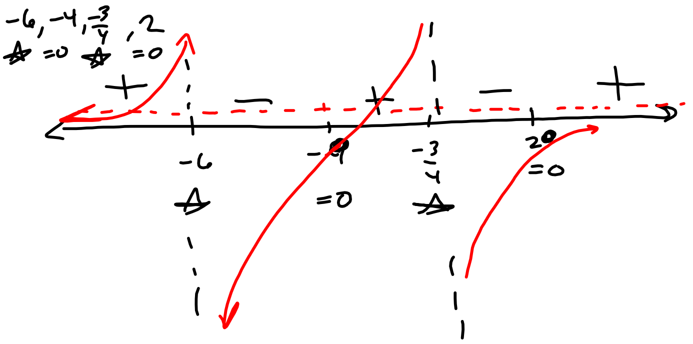
$$R(x) = \frac{3x^2 + 6x - 24}{4x^2 + 27x + 18} = \frac{(3x - 6)(x + 4)}{(x+6)(4x+3)}$$





$$\frac{3x^2+6x-24}{4x^2+27x+18} = \frac{\cancel{x^2}\left(3+\frac{6}{x}-\frac{24}{x^2}\right)}{\cancel{x^2}\left(4+\frac{27}{x}+\frac{18}{x^2}\right)} \xrightarrow{x \to \pm\infty} \frac{3}{4}$$

$x \longrightarrow \pm\infty$ is kind of like $|x| \longrightarrow \infty$



$$\frac{3x^2+6x-24}{4x^2+27x+18} = R(x) = \frac{u}{v} \longrightarrow R' = \left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$$

$$= \frac{(6x+6)(4x^2+27x+18)-(3x^2+6x-24)(8x+27)}{(4x^2+27x+18)^2}$$

This is as far as you'd go on a test question asking for R'' under a time control.

We were running out of time at the end of class, so we went to the technology crutch.

Continuing...

$$\frac{(6x+6)(4x^2+27x+18)-(3x^2+6x-24)(8x+27)}{(4x^2+27x+18)^2}$$

$$= \frac{6(x+1)(4x^2+27x+18)-(3x^2+6x-24)(8x+27)}{(4x^2+27x+18)^2}$$

$27 \cdot 24 = 108 + 540 = 648$

$$\frac{6(4x^3+27x^2+18x+4x^2+27x+18)-(24x^3+81x^2+48x^2+162x-192x-648)}{(4x+3)^2(x+6)^2}$$

$$= \frac{24x^3+162x^2+108x+24x^2+162x+108-(24x^3+129x^2-30x-648)}{((4x+3)(x+6))^2}$$

$$= \frac{24x^3+186x^2+270x+108-24x^3-129x^2+30x+648}{(\quad)^2}$$

$4 \cdot 756 = 228$, 6014, 15120, 151200, 162334

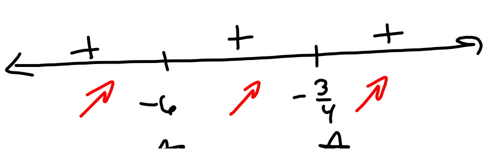
$$= \frac{57x^2+300x+756}{(\quad)^2} \text{ set } = 0 \longrightarrow$$

$ax^2+bx+c=0$, where $a=57$, $b=300$, $c=756$

$\longrightarrow b^2-4ac = 300^2-4(57)(756) = 90000-(228)(756)$

$= 90000 - 162334 < 0 \longrightarrow$ No real zeros. Just

Denom $= 0$ @ $-\frac{3}{4}, -6$

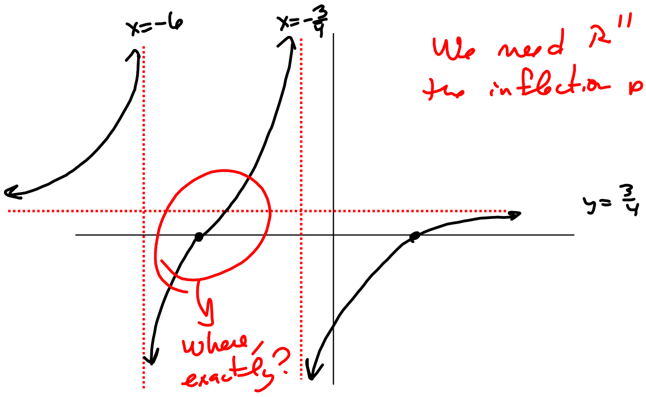


$(\quad)^2$ doesn't change

The sign pattern for R' is totally in agreement with the College Algebra.

What the College Algebra can't do is find the inflection point that must exist, because (look and think) is has to approach + infinity as x -> -3/4 from the left and it has to approach - infinity as it approaches x = - 6 from the right.

Somewhere in between it goes from ☹ to ☺



We need R'' to find the inflection point (mood change)

$$\frac{57x^2+300x+756}{((x+6)(4x+3))^2} = R'(x)$$

$$R''(x) = \frac{(114x+300)((x+6)(4x+3))^2}{(((x+6)(4x+3))^2)^2}$$

$$-\frac{(57x^2+300x+756)(2((x+6)(4x+3))^1(8x+27))}{(((x+6)(4x+3))^2)^2}$$

$$= \frac{(x+6)(4x+3)\left((114x+300)(x+6)(4x+3) - (57x^2+300x+756)(8x+27)\right)}{((x+6)(4x+3))^4}$$

$$= \frac{(114x+300)(4x^2+27x+18) - (8x+27)(57x^2+300x+756)}{((x+6)(4x+3))^3}$$

$$= \frac{456x^3+3078x^2+2052x+1200x^2+8100x+5400}{(\quad)^3}$$

$$+ \frac{-456x^3-2400x^2-6048x-1539x^2-8100x-20412}{(\quad)^3}$$

$$= \frac{4278x^2+10152x+5400-3939x^2-14148x-20412}{(\quad)^3}$$

$$= \frac{339x^2+6213x-15012}{(x+6)^3(4x+3)^3}$$

57×27: 399, 1140, 1539

$756 \times 8 = 6048$

$57 \times 8 = 456$

114×18: 912, 1140, 2052

756×27: 5292, 15120, 20412

114×27: 798, 2280, 3078

$R''(x) = \cdots$

$R''(x) = 0 \Rightarrow x \approx$ -3.121026351

& Remember the denominator has same zeros as R & R'. Just check the multiplicities of them to know if you have a sign change.

-6 & $-\frac{3}{4}$ are the zeros

$$\frac{-456x^3 - 3000x^2 - 18144x - 35424}{(x-6)^3(4x+3)^3}$$