2410

$$f(x) = 2x^3 - 6x^2 - 90x \longrightarrow f'(x) = 6x^2 - 12x - 90$$

a. (5 pts) Convince me that there is a point $c \in [1,10]$ such that $f'(c) = 66$, *without finding c, itself!*

Looks like MVT

$\frac{f(10)-f(1)}{10-1} = f'(c)$ for some $c \in (1,10)$.

$\frac{f(10)-f(1)}{10-1} = -2300$

$$\begin{array}{r|rrrr} 10 & 2 & -6 & -90 & 0 \\ & & 20 & -140 & -2300 \\ \hline & 2 & -14 & -230 & -2300 \end{array}$$

If I could execute this, I would find $\frac{f(b)-f(a)}{b-a} = 66$

Then apply MVT. Find c.

b. (5 pts) Find *c*.

$\longrightarrow f'(x) = 6x^2 - 12x - 90 \overset{\text{SET}}{=} 0 \longrightarrow$

$x^2 - 2x - 15 = (x-5)(x+3) = 0 \longrightarrow$

$x = -3, 5$

So, $c = 5$ does it.

2. Let $f(x) = (x+5)^3 (x-6)^2$.

a. (5 pts) Find the absolute maximum and minimum of f on the interval $[-3, 0]$.

$f(-3) = (-3+5)^3(-3-6)^2 = 2^3(-9)^2 = 8(81) = 648$

$f(0) = 5^3(-6)^2 = 125(36) = 3250$

$f'(x) = 3(x+5)^2(1)(x-6)^2 + (x+5)^3 2(x-6)^1(1)$

$= 3(x+5)^2(x-6)^2 + 2(x+5)^3(x-6)$ SET 0

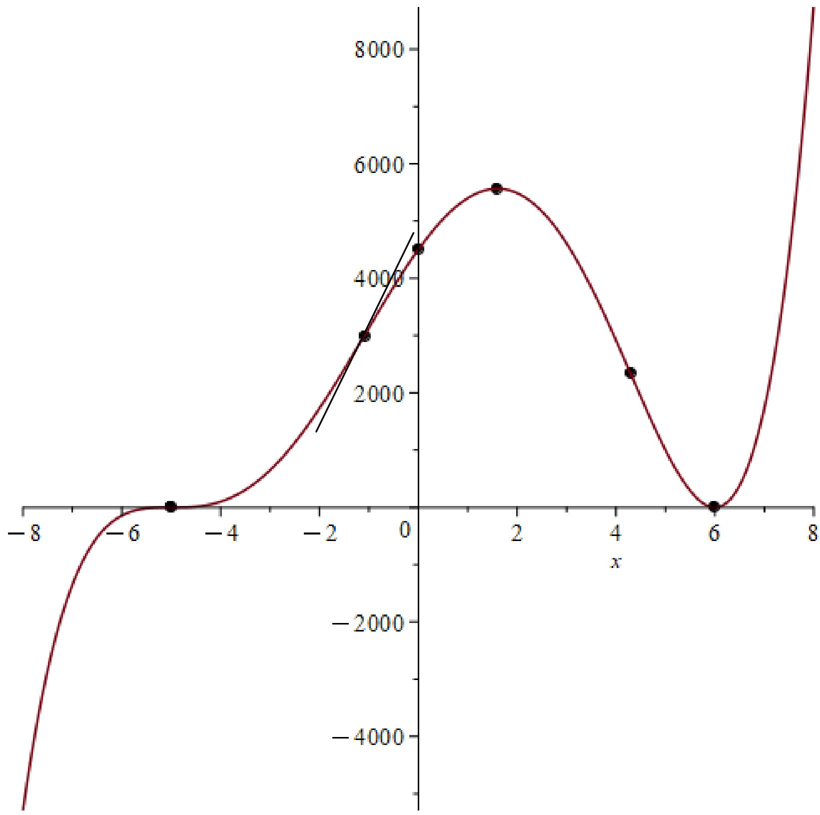
$\Rightarrow (x+5)^2(x-6)\left(3(x-6) + 2(x+5)\right)$

$= (x+5)^2(x-6)(3x-18+2x+10)$

$= (x+5)^2(x-6)(5x-8) = 0$ $\rightarrow$ only one between -3 & 0

$\Rightarrow x \in \{-5, \frac{8}{5}, 6\}$

$f\left(\frac{8}{5}\right)$

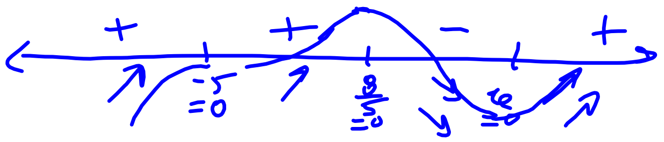

8000
6000
4000
2000
−2000
−4000
−8
−6
−4
−2
0
2
4
6
8
x

b. (5 pts) Find the *open* intervals on which f is increasing. Find the open intervals on which f is decreasing.

$f\left(\frac{8}{5}\right)$ is the main candidate

$= 5565.922560$ Local Max

$\frac{8}{5}, 6, -5$



$\left(-\infty, \frac{8}{5}\right) \cup (6, \infty)$

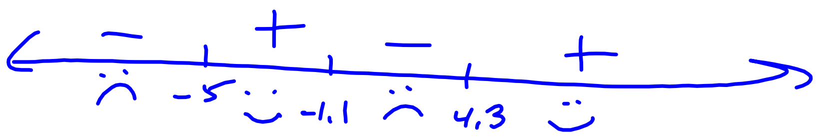
WebAssign accepts $\left(-\infty, \frac{8}{5}\right), (6, \infty)$, but I don't

c. (5 pts) Find the open intervals on which f is concave up. Find the open intervals on which f is concave down.

$f''(x) = 0$ @ $x = -5$ & $\frac{16 \pm 11\sqrt{6}}{10}$ ≈ 4.3, ≈ -1.1

$f''(x) = 2(x+5)(10x^2 - 32x - 47)$

$-1.1, -4.3, -5$



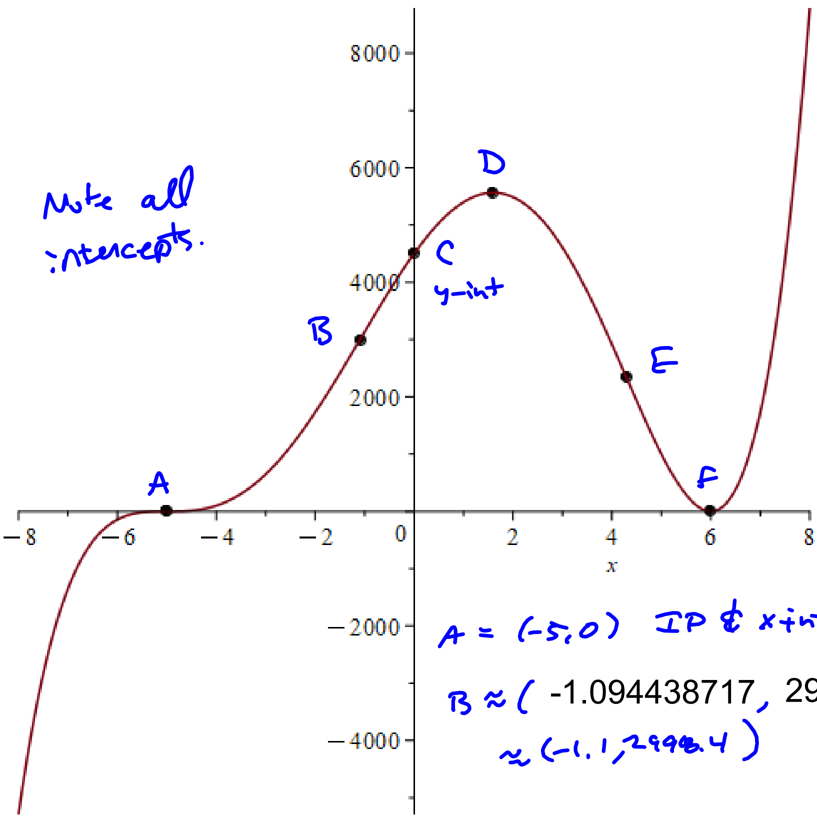
Concave up: $\left(-5, \frac{16 - 11\sqrt{6}}{10}\right) \cup \left(\frac{16 + 11\sqrt{6}}{10}, \infty\right)$

Concave down $(-\infty, -5) \cup \left(\frac{16 - 11\sqrt{6}}{10}, \frac{16 + 11\sqrt{6}}{10}\right)$

Inflection points @ $x = -5, \frac{16 \pm 11\sqrt{6}}{10}$

d. (5 pts) Use all the information from parts a – d to sketch the graph of f. Label all intercepts, max/min points, and inflection points. You may put the ordered-pair labels directly on the graph or make a legend/key as I will demonstrate in lecture.

Note all intercepts.



A = (-5,0) IP & x-int

B ≈ (-1.094438717, 2998.378489) I.P.

≈ (-1.1, 2998.4)

3. Let $f(x)=(x+2)^2\sqrt{16-x^2}$.

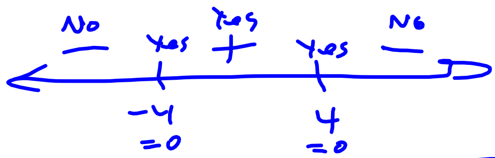
a. (5 pts) What is the domain of f?

Need $16-x^2 \ge 0$

$(4-x)(4+x) \ge 0$

$-x^2$

≥ 0



$\rightarrow$ $x \in \boxed{[-4,4] = D(f)}$

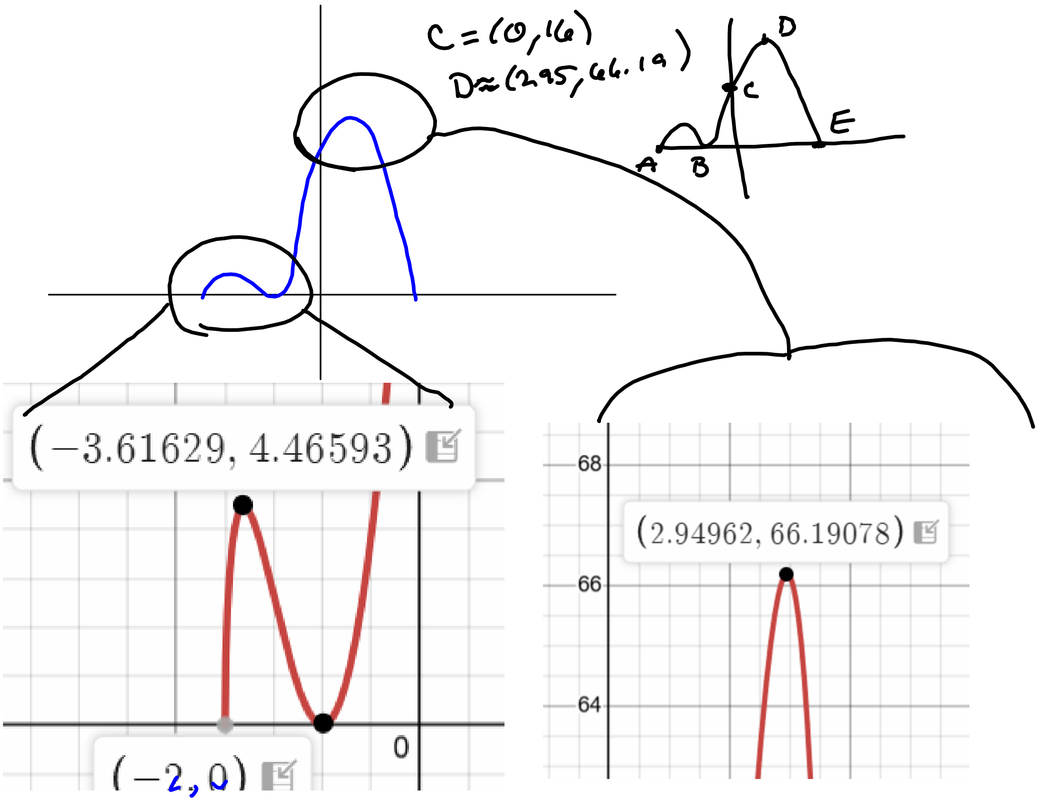
b. (5 pts) Use a graphing utility to sketch the graph of f. Include all max/min values and intercepts. Round answers to 2 decimal places.

$A \approx (-3.62, 4.47)$

$B = (-2, 0)$

$C = (0, 16)$

$D \approx (2.95, 66.19)$



c. (**Bonus 5 pts**) Use calculus to find the *exact* maximum value. What is the range of f?

$$(x+2)^2\sqrt{16-x^2} = f(x) = (x+2)^2(16-x^2)^{\frac{1}{2}}$$

$$\Longrightarrow f'(x) = 2(x+2)^1(1)(16-x^2)^{\frac{1}{2}} + (x+2)^2\left(\frac{1}{2}(16-x^2)^{-\frac{1}{2}}(-2x)\right)$$

$$= \frac{2(x+2)(16-x^2)^{\frac{1}{2}}}{1}\cdot\frac{(16-x^2)^{\frac{1}{2}}}{(16-x^2)^{\frac{1}{2}}} - \frac{x(x+2)^2}{(16-x^2)^{\frac{1}{2}}}$$

$$= \frac{2(x+2)(16-x^2)^{\frac{1}{2}} - x(x+2)^2}{(16-x^2)^{\frac{1}{2}}} = \frac{(x+2)\left(2(16-x^2) - x(x+2)\right)}{\text{same}}$$

$$= \frac{(x+2)(32-2x^2-x^2-2x)}{\text{same}} = \frac{(x+2)(-3x^2-2x+32)}{(16-x^2)^{\frac{1}{2}}} \overset{\text{SET}}{=} 0$$

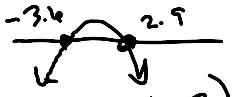
$$3x^2+2x-22 = 3\left(x^2+\frac{2}{3}x+\left(\frac{1}{3}\right)^2-\frac{1}{9}-\frac{22}{3}\cdot\frac{3}{3}\right)$$

$$= 3\left(\left(x+\frac{1}{3}\right)^2-\frac{67}{9}\right) = 3\left(x+\frac{1}{3}\right)^2 - \frac{67}{3} \overset{\text{SET}}{=} 0$$

97?

$$\left(x+\frac{1}{3}\right)^2 = \frac{67}{9}$$

$$x = \frac{-1\pm\sqrt{67}}{3}$$



$$f'(x) = \frac{(x+2)(-3x^2-2x+32)}{(16-x^2)^{\frac{1}{2}}}$$

$f' = 0$

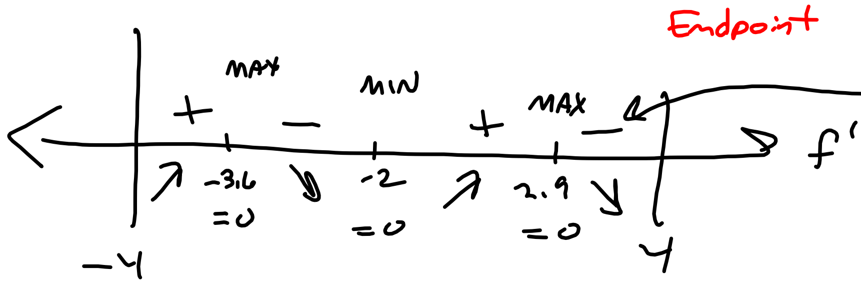
$x = -2,$

$x \approx -3.6163$

$x \approx 2.9496$

f' $\cancel{\nexists}$ on boundary

Endpoint



$$(x+2)(-3)\left(x-\left(\frac{1+\sqrt{97}}{3}\right)\right)\left(x-\left(\frac{1-\sqrt{97}}{3}\right)\right)$$

Test: $x = -1$ or $x = 3$

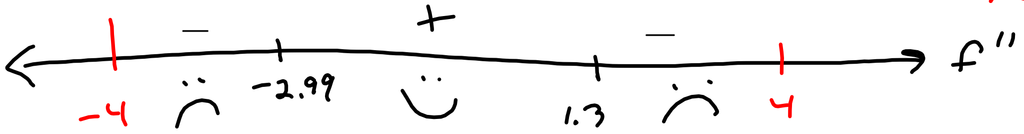
$$\frac{(-3+2)(-3(3)^2-2(-3)+22)}{+} = \frac{-(-27+6+22)}{+} = \frac{-(+)}{+}$$

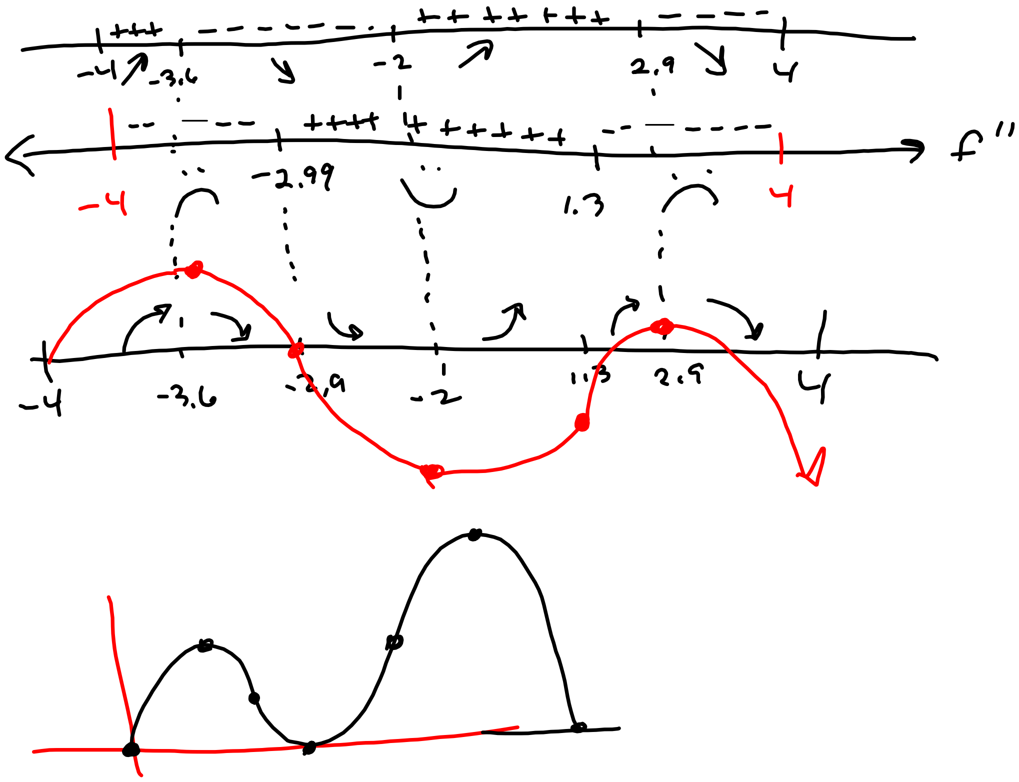
$= -$

$$f''(x) = \frac{6x^4 + 8x^3 - 144x^2 - 192x + 448}{(16 - x^2)^{3/2}} \underset{=}{\text{SET}}\ 0 \longrightarrow$$

$x \approx -4.2327,\ -2.9917,\ 1.27829,\ 4.6128$

$\notin D$

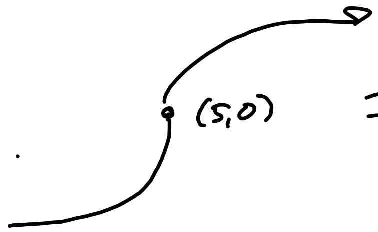



-4
-3.6
-2
2.9
4
f''
-2.99
1.3
-3.6
-2.9
-2
1.3
2.9
4
-4

4. (5 pts) Let $f(x) = x(x-5)^{\frac{5}{7}}$. Sketch the graph of f. Clearly label all x- and y-intercepts, local max/min points, and inflection points. Each label should be an ordered pair or a letter referring to an ordered pair in a key or legend for the sketch. It's vital that your sketch capture the main features and shape.

$(x-5)^{5/7}$ "=" $(x-5)^{\frac{1}{3}}$

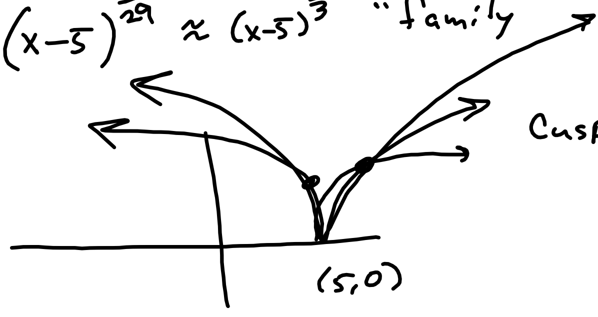
$x^{\frac{odd}{ODD}}$



IP @ $x=5$

$x^{\frac{even}{odd}}$ & ODD > even

$(x-5)^{\frac{6}{29}} \approx (x-5)^{\frac{2}{3}}$ "family



Cusp & min @ $x=5$