

2416

$$f(x) = 2x^3 - 6x^2 - 90x \rightarrow f'(x) = 6x^2 - 12x - 90$$

- a. (5 pts) Convince me that there is a point $c \in [1, 10]$ such that $f'(c) = 66$, without finding c , itself!

Looks like MVT

$$\frac{f(10) - f(1)}{10 - 1} = f'(c) \text{ for some } c \in (1, 10).$$

$$\frac{f(10) - f(1)}{10 - 1} = -2300$$

$$\begin{array}{r} 10 \overline{) 2 \quad -6 \quad -90 \quad 0} \\ \underline{20 \quad -140 \quad -2300} \\ 2 \quad -14 \quad -230 \quad -2300 \end{array}$$

If I could execute this, I would

$$\text{find } \frac{f(b) - f(a)}{b - a} = 66$$

Then apply MVT. Find c .

- b. (5 pts) Find c .

$$\rightarrow f'(x) = 6x^2 - 12x - 90 \stackrel{!}{=} 0 \rightarrow$$

$$x^2 - 2x - 15 = (x - 5)(x + 3) = 0 \rightarrow$$

$$x = -3, 5$$

So, $c = 5$ does it.

2. Let $f(x) = (x+5)^3(x-6)^2$.

a. (5 pts) Find the absolute maximum and minimum of f on the interval $[-3, 0]$.

$$f(-3) = (-3+5)^3(-3-6)^2 = 2^3(-9)^2 = 8(81) = 648$$

$$f(0) = 5^3(-6)^2 = 125(36) = 4500$$

$$f'(x) = 3(x+5)^2(1)(x-6)^2 + (x+5)^3 \cdot 2(x-6)(1)$$

$$= 3(x+5)^2(x-6)^2 + 2(x+5)^3(x-6) \stackrel{S \subseteq I}{=} 0$$

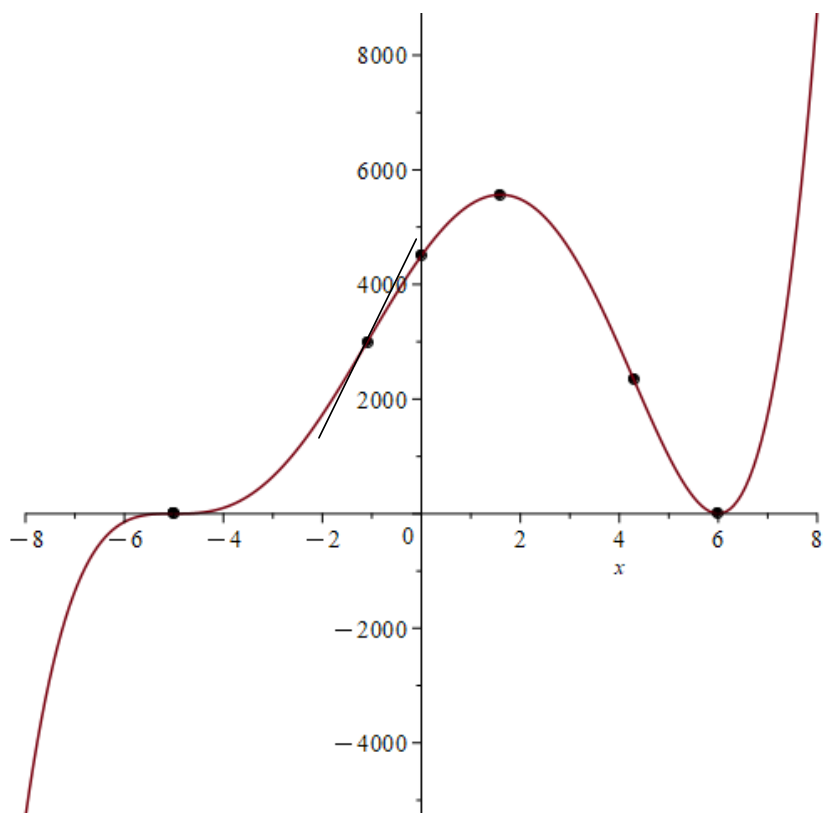
$$\rightarrow (x+5)^2(x-6)(3(x-6) + 2(x+5))$$

$$= (x+5)^2(x-6)(3x-18+2x+10)$$

$$= (x+5)^2(x-6)(5x-8) = 0$$

$$\rightarrow x \in \{-5, \frac{8}{5}, 6\} \rightarrow \text{only one between } -3 \text{ \& } 0$$

$$f\left(\frac{8}{5}\right)$$



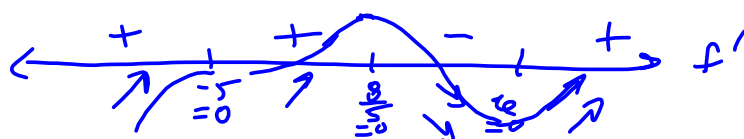
- b. (5 pts) Find the open intervals on which f is increasing. Find the open intervals on which f is decreasing.

$f(\frac{8}{5})$ is the main candidate

$$= 5565.922560$$

Local Max

$$\frac{8}{5}, 4, -5$$



$$(-\infty, \frac{8}{5}) \cup (6, \infty)$$

WebAssign accepts $(-\infty, \frac{8}{5}) \cup (4, \infty)$, but I don't

- c. (5 pts) Find the open intervals on which f is concave up. Find the open intervals on which f is concave down.

$$f''(x) = 0 \quad \text{a) } x = -5 \quad \text{b) } \frac{16 \pm 11\sqrt{6}}{10} \quad \begin{matrix} \nearrow \approx 4.3 \\ \searrow \approx -1.1 \end{matrix}$$

$$f''(x) = 2(x+5)(10x^2 - 32x - 47)$$

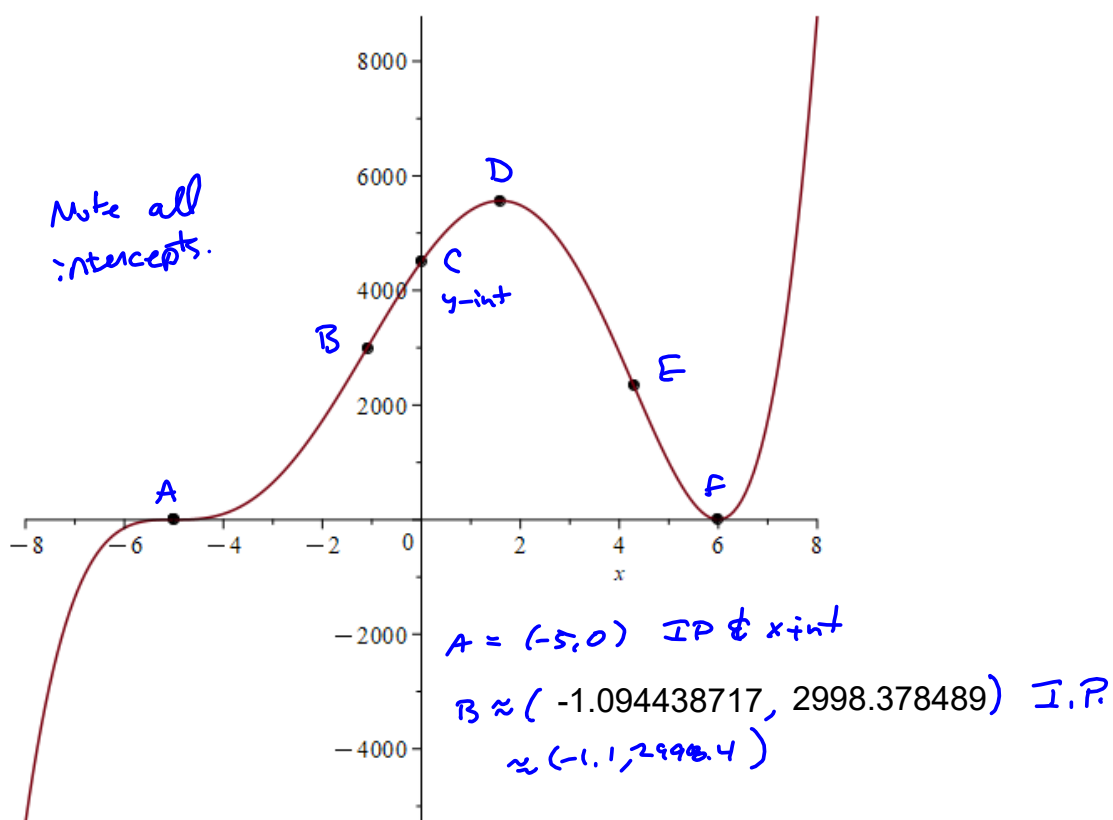


$$\text{Concave up: } (-5, \frac{16-11\sqrt{6}}{10}) \cup (\frac{16+11\sqrt{6}}{10}, \infty)$$

$$\text{Concave down } (-\infty, -5) \cup (\frac{16-11\sqrt{6}}{10}, \frac{16+11\sqrt{6}}{10})$$

$$\text{Inflection points a) } x = -5, \frac{16 \pm 11\sqrt{6}}{10}$$

- d. (5 pts) Use all the information from parts a – d to sketch the graph of f . Label all intercepts, max/min points, and inflection points. You may put the ordered-pair labels directly on the graph or make a legend/key as I will demonstrate in lecture.



3. Let $f(x) = (x+2)^2 \sqrt{16-x^2}$.

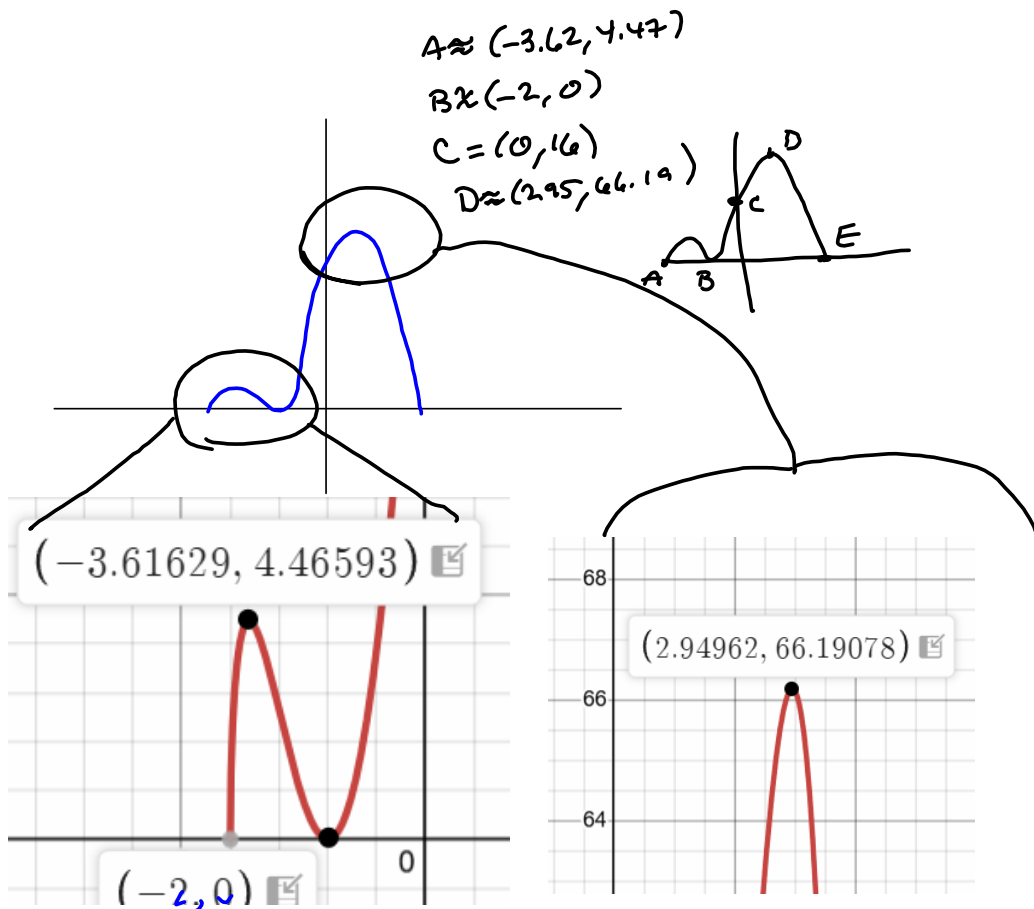
a. (5 pts) What is the domain of f ?

Need $16-x^2 \geq 0$

$$(4-x)(4+x) \geq 0$$

$x \in [-4, 4] = D(f)$

b. (5 pts) Use a graphing utility to sketch the graph of f . Include all max/min values and intercepts. Round answers to 2 decimal places.



∴ (Bonus 5 pts) Use calculus to find the *exact* maximum value. What is the range of f ?

$$(x+2)^2 \sqrt{16-x^2} = f(x) = (x+2)^2 (16-x^2)^{\frac{1}{2}}$$

$$\rightarrow f'(x) = 2(x+2)(1)(16-x^2)^{\frac{1}{2}} + (x+2)^2 \left(\frac{1}{2} (16-x^2)^{-\frac{1}{2}} (-2x) \right)$$

$$= \frac{2(x+2)(16-x^2)^{\frac{1}{2}}}{1} \cdot \frac{(16-x^2)^{\frac{1}{2}}}{(16-x^2)^{\frac{1}{2}}} - \frac{x(x+2)^2}{(16-x^2)^{\frac{1}{2}}}$$

$$= \frac{2(x+2)(16-x^2) - x(x+2)^2}{(16-x^2)^{\frac{1}{2}}} = \frac{(x+2)(2(16-x^2) - x(x+2))}{\text{same}}$$

$$= \frac{(x+2)(32-2x^2-x^2-2x)}{\text{same}} = \frac{(x+2)(-3x^2-2x+32)}{(16-x^2)^{\frac{1}{2}}} \quad \underline{\underline{\text{SET } 0}}$$

4. (5 pts) Let $f(x) = x(x-5)^{\frac{5}{7}}$. Sketch the graph of f . Clearly label all x - and y -intercepts, local max/min points, and inflection points. Each label should be an ordered pair or a letter referring to an ordered pair in a key or legend for the sketch. It's vital that your sketch capture the main features and shape.