$with(plots)$:

#1 Mean Value Theorem

$with(plots)$:

$fp := x \to 6 \cdot (x + 3) \cdot (x - 5)$

$$fp := x \mapsto (6 \cdot x + 18) \cdot (x - 5) \tag{1.1}$$

$\int fp(x)\, dx$

$$2\,x^3 - 6\,x^2 - 90\,x \tag{1.2}$$

$f := x \to 2\,x^3 - 6\,x^2 - 90\,x$

$$f := x \mapsto 2 \cdot x^3 - 6 \cdot x^2 - 90 \cdot x \tag{1.3}$$

$factor(f(x))$

$$2\,x\left(x^2 - 3\,x - 45\right) \tag{1.4}$$

$a := 1; b := 10$

$$a := 1$$
$$b := 10 \tag{1.5}$$

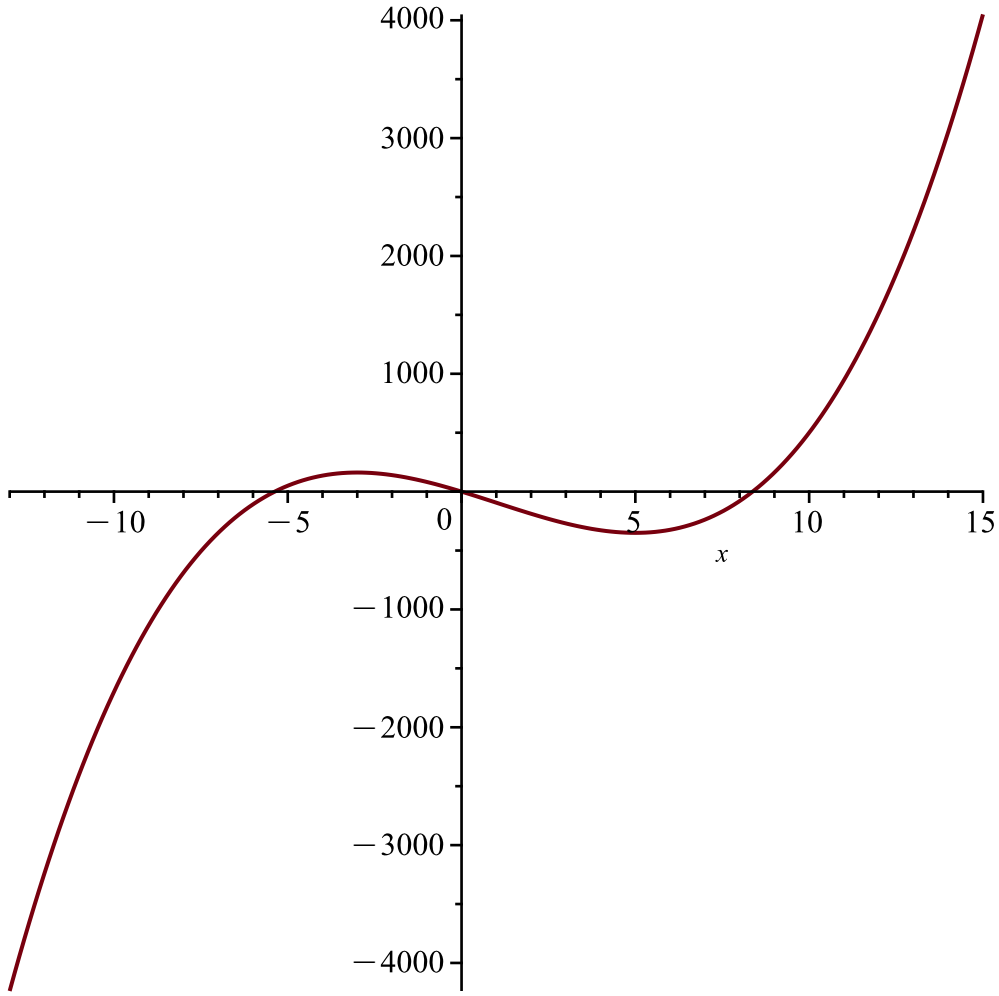
$\frac{(f(b) - f(a))}{b - a}$

$$66 \tag{1.6}$$

$f(10)$

$$500 \tag{1.7}$$

$plot(f(x))$



$fp := \mathrm{D}(f)$

$$fp := x \mapsto 6 \cdot x^2 - 12 \cdot x - 90 \quad \textbf{(1.8)}$$

$factor(fp(x))$

$$6\,(x+3)\,(x-5) \quad \textbf{(1.9)}$$

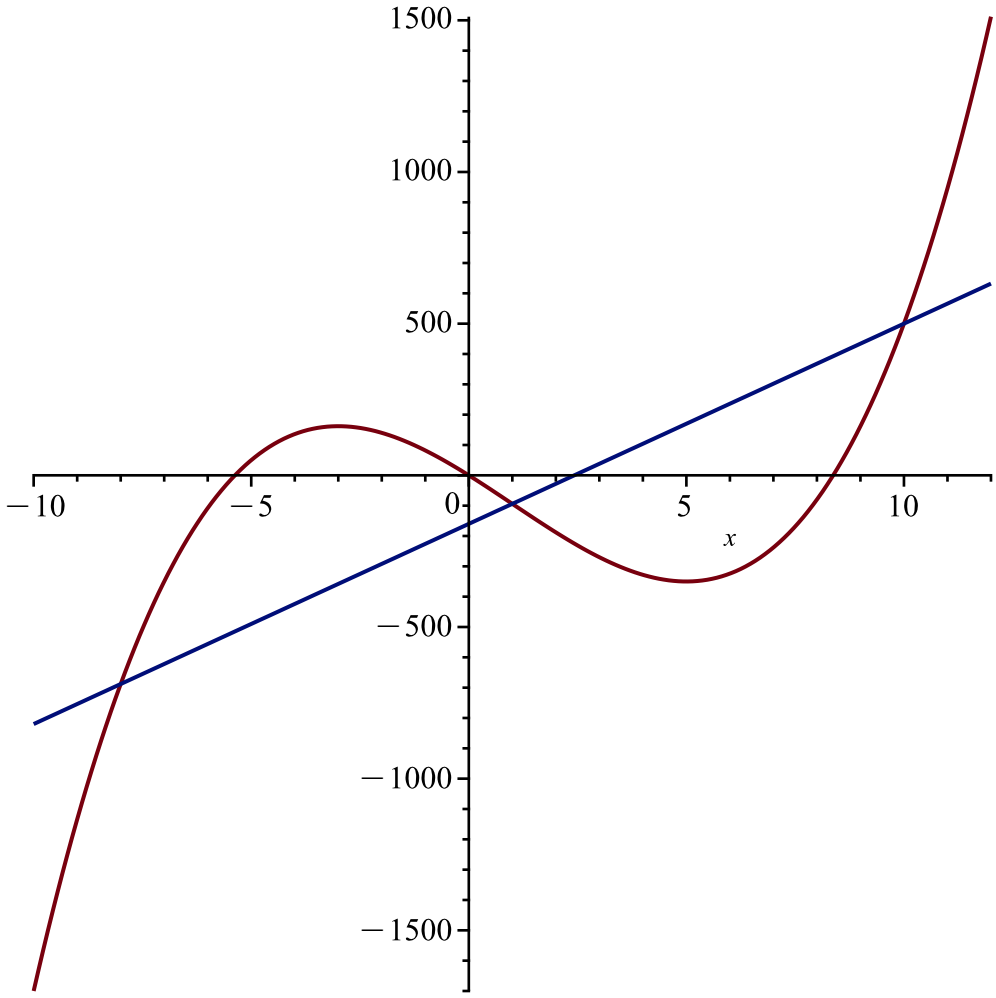
$fp\left(\frac{8}{5}\right)$

$$-\frac{2346}{25} \quad \textbf{(1.10)}$$

$$\frac{(f(b)-f(a))}{b-a}$$

$$66 \quad \textbf{(1.11)}$$

$plot([f(x), secantline(x)], x = -10..12)$



$solve\left(fp(x) = \frac{(f(b) - f(a))}{b - a} \right)$

$$1 + 3\sqrt{3},\ 1 - 3\sqrt{3} \tag{1.12}$$

$fp\left(1 + 3\sqrt{3}\right)$

$$6\left(1 + 3\sqrt{3}\right)^2 - 102 - 36\sqrt{3} \tag{1.13}$$

$expand(\%)$

$$66 \tag{1.14}$$

$L1 := x \to fp\left(1 + 3\sqrt{3}\right) \cdot \left(x - \left(1 + 3\sqrt{3}\right)\right) + f\left(1 + 3\sqrt{3}\right)$

$$L1 := x \mapsto fp\left(1 + 3 \cdot \sqrt{3}\right) \cdot \left(x - 1 - 3 \cdot \sqrt{3}\right) + f\left(1 + 3 \cdot \sqrt{3}\right) \tag{1.15}$$

$L1(x)$

$$\left(6\left(1 + 3\sqrt{3}\right)^2 - 102 - 36\sqrt{3}\right)\left(x - 1 - 3\sqrt{3}\right) + 2\left(1 + 3\sqrt{3}\right)^3 - 6\left(1 + 3\sqrt{3}\right)^2 - 90 - 270\sqrt{3} \tag{1.16}$$

$expand(\%)$

$$66x - 160 - 324\sqrt{3} \tag{1.17}$$

$fp\left(1 + 3\sqrt{3}\right)$

$$6\left(1+3\sqrt{3}\right)^2-102-36\sqrt{3} \tag{1.18}$$

$expand(\%)$

$$66 \tag{1.19}$$

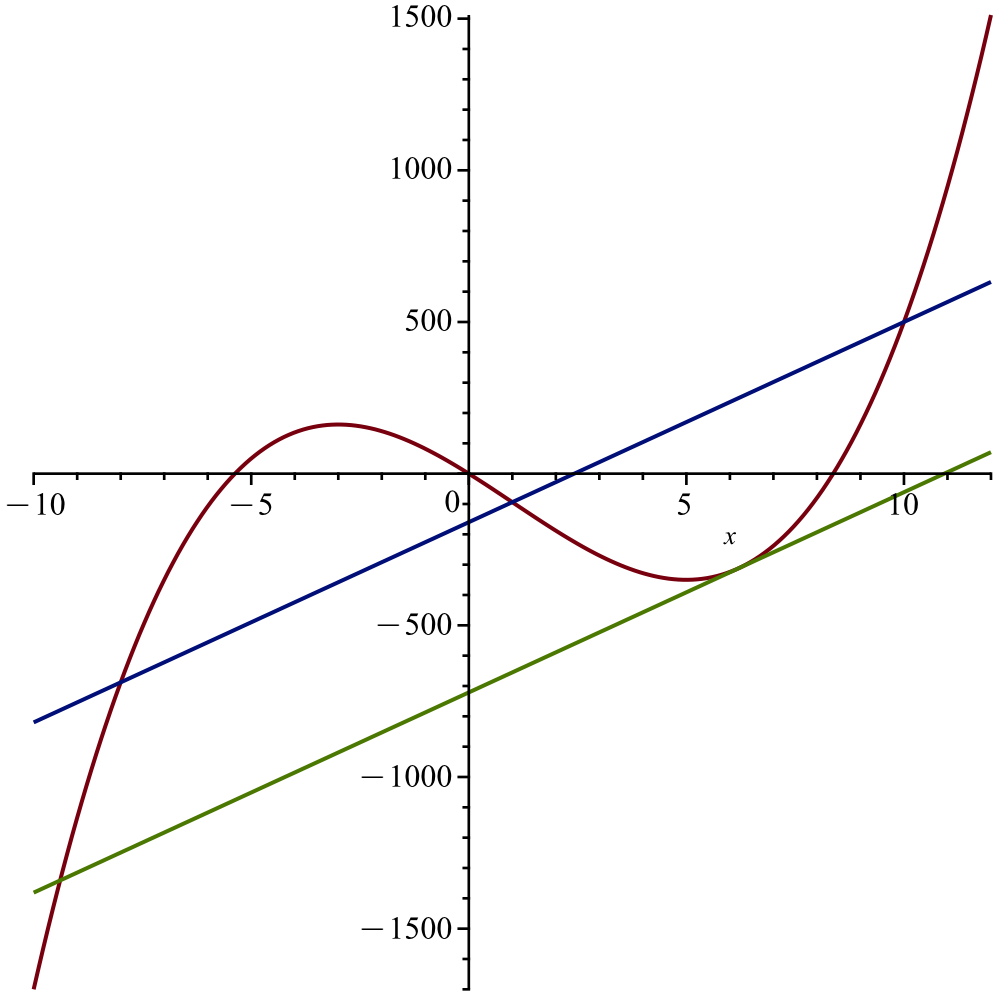
$f\left(1+\sqrt{3}\right)$

$$2\left(1+\sqrt{3}\right)^3-6\left(1+\sqrt{3}\right)^2-90-90\sqrt{3} \tag{1.20}$$

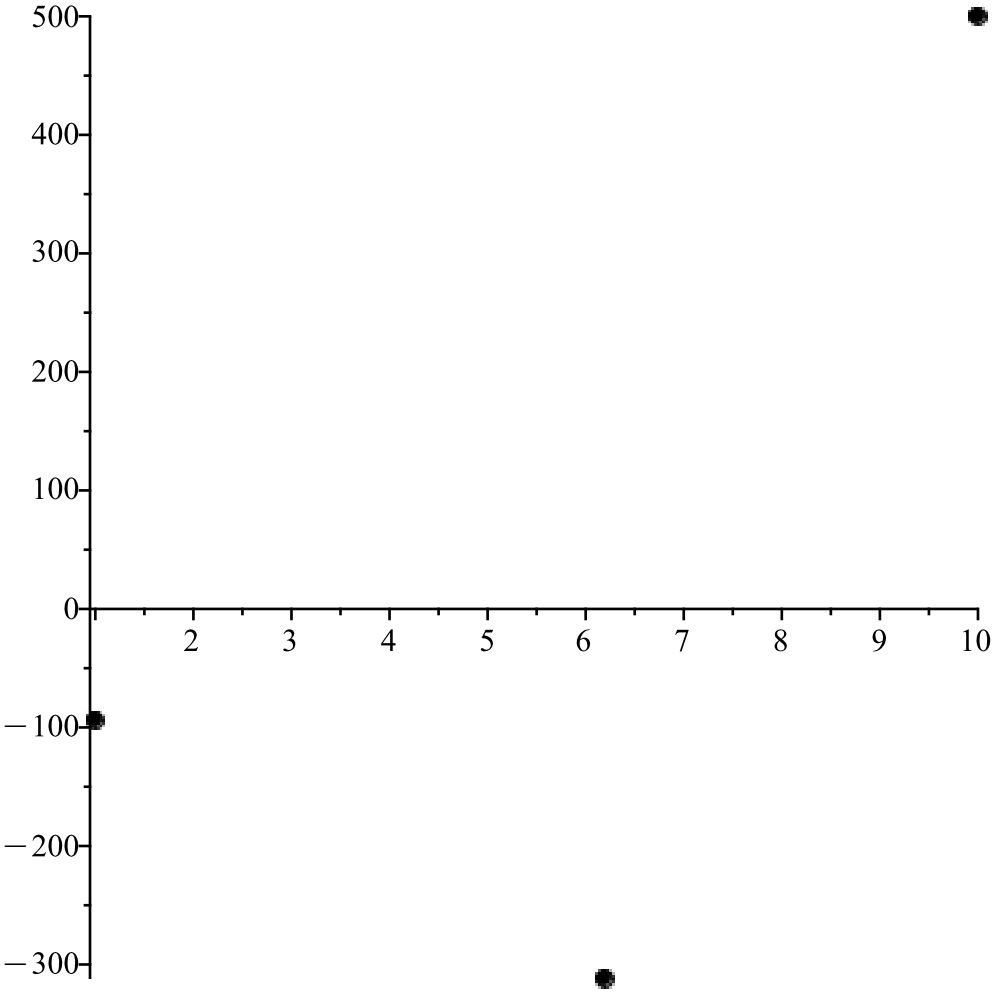
$expand(\%)$

$$-94-90\sqrt{3} \tag{1.21}$$

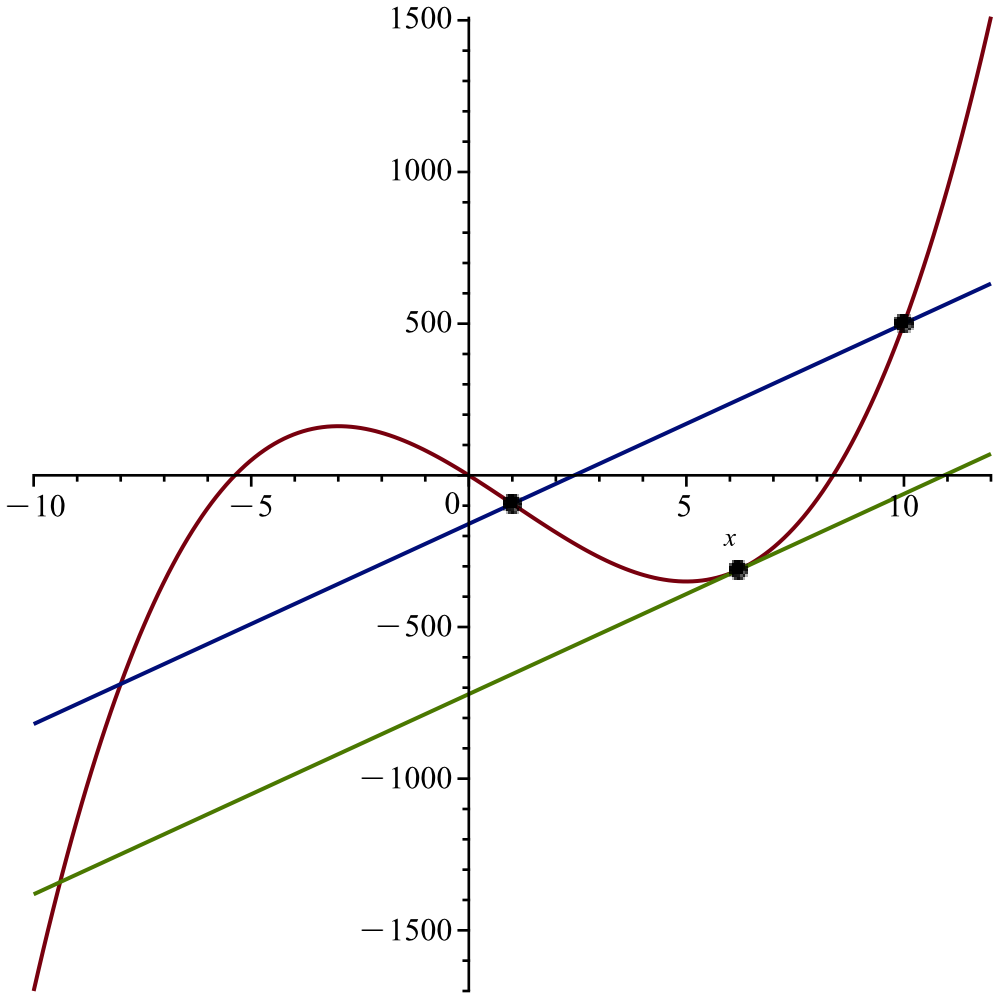
$fplots := plot([f(x), secantline(x), L1(x)], x=-10..12)$



$mypoints := pointplot\left([[1, f(1)], \left[1+3\sqrt{3}, f\left(1+3\sqrt{3}\right)\right], [10, f(10)]], symbolsize=12, symbol=solidcircle\right)$



display([*fplots*, *mypoints*])



#2

$f := x \to (x+5)^3 \cdot (x-6)^2$

$$f := x \mapsto (x+5)^3 \cdot (x-6)^2 \tag{2.1}$$

$fp := \mathrm{D}(f)$

$$fp := x \mapsto 3 \cdot (x+5)^2 \cdot (x-6)^2 + 2 \cdot (x+5)^3 \cdot (x-6) \tag{2.2}$$

$f\left(\frac{8}{5}\right)$

$$\frac{17393508}{3125} \tag{2.3}$$

$evalf(\%)$

$$5565.922560 \tag{2.4}$$

$factor(fp(x))$

$$(x+5)^2 (x-6) (5x-8) \tag{2.5}$$

$fpp := \mathrm{D}(fp)$

$$fpp := x \mapsto 6 \cdot (x+5) \cdot (x-6)^2 + 12 \cdot (x+5)^2 \cdot (x-6) + 2 \cdot (x+5)^3 \tag{2.6}$$

$factor(fpp(x))$

$$2\,(x+5)\,\left(10\,x^2-32\,x-47\right) \tag{2.7}$$

$solve(fpp(x)=0)$

$$-5,\ \frac{8}{5}+\frac{11\sqrt{6}}{10},\ \frac{8}{5}-\frac{11\sqrt{6}}{10} \tag{2.8}$$

$evalf\left(\frac{8}{5}-\frac{11\sqrt{6}}{10}\right)$

$$-1.094438717 \tag{2.9}$$

$$pointslist := \left[[-5,f(-5)],\,[6,f(6)],\,\left[\frac{8}{5},f\left(\frac{8}{5}\right)\right],\,[0,f(0)],\,[-5,f(-5)],\,\left[\frac{8}{5}+\frac{11\sqrt{6}}{10},\,f\left(\frac{8}{5}+\frac{11\sqrt{6}}{10}\right)\right],\,\left[\frac{8}{5}-\frac{11\sqrt{6}}{10},f\left(\frac{8}{5}-\frac{11\sqrt{6}}{10}\right)\right]\right]$$

$$pointslist := \left[[-5,0],\,[6,0],\,\left[\frac{8}{5},\frac{17393508}{3125}\right],\,[0,4500],\,[-5,0],\,\left[\frac{8}{5}+\frac{11\sqrt{6}}{10},\left(\frac{33}{5}+\frac{11\sqrt{6}}{10}\right)^3\left(-\frac{22}{5}+\frac{11\sqrt{6}}{10}\right)^2\right],\,\left[\frac{8}{5}-\frac{11\sqrt{6}}{10},\left(\frac{33}{5}-\frac{11\sqrt{6}}{10}\right)^3\left(-\frac{22}{5}-\frac{11\sqrt{6}}{10}\right)^2\right]\right] \tag{2.10}$$

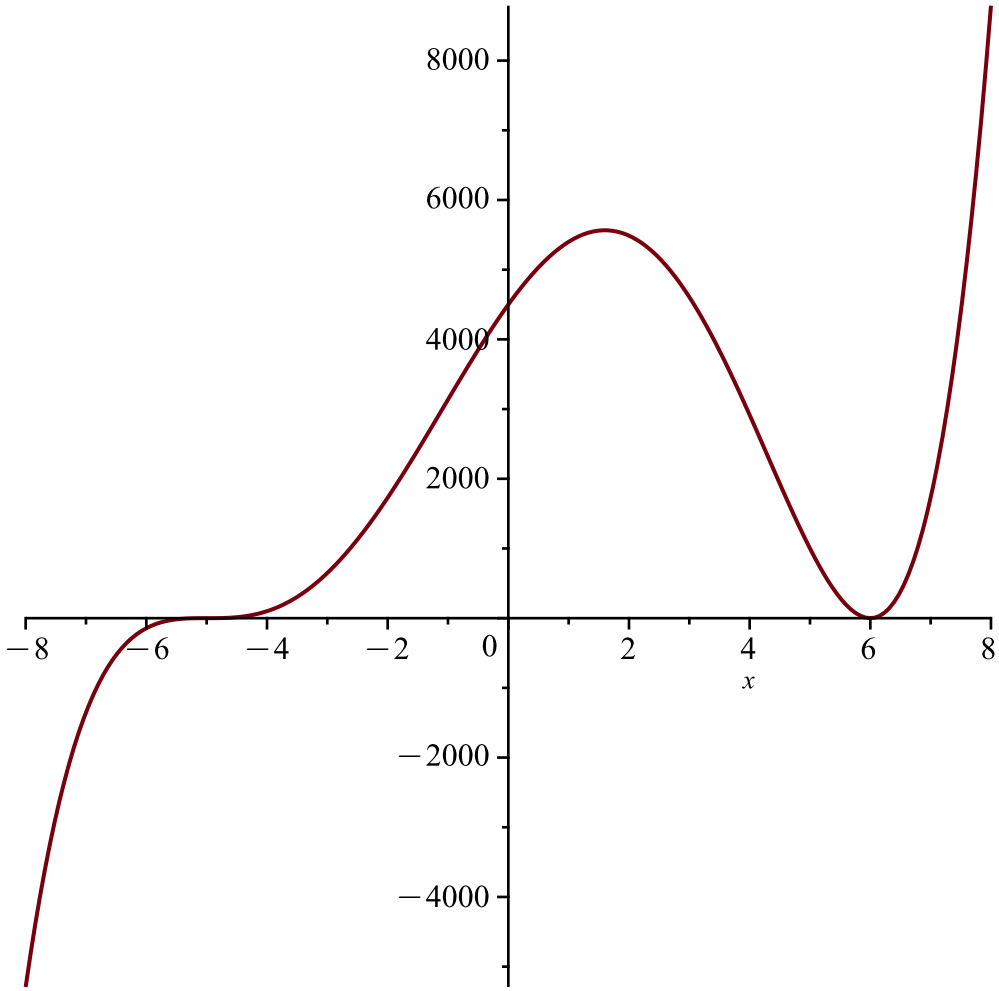
$evalf(\%)$

$$[[-5.,0.],\,[6.,0.],\,[1.600000000,5565.922560],\,[0.,4500.],\,[-5.,0.],\,[4.294438717,2335.630631],\,[-1.094438717,2998.378489]] \tag{2.11}$$

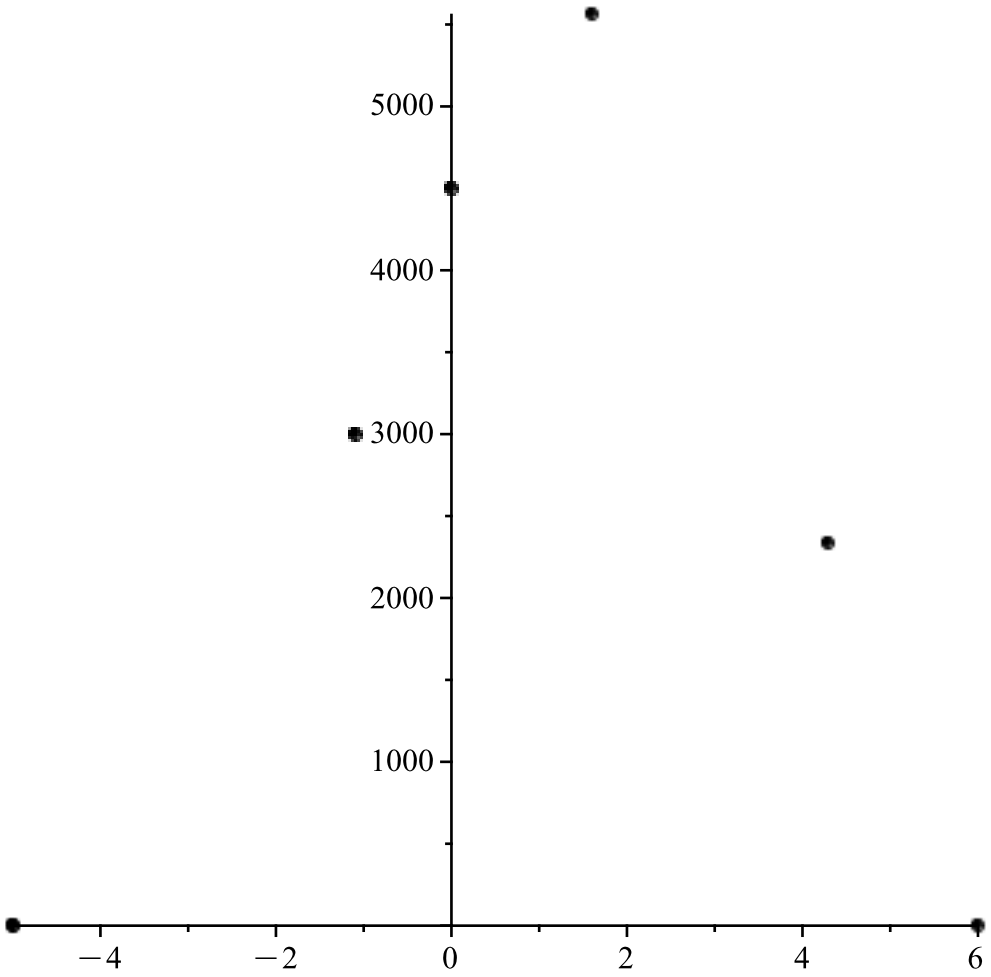
$$pointslist := \left[[-5,f(-5)],\,[6,f(6)],\,\left[\frac{8}{5},f\left(\frac{8}{5}\right)\right],\,[0,f(0)],\,[-5,f(-5)],\,\left[\frac{8}{5}+\frac{11\sqrt{6}}{10},\,f\left(\frac{8}{5}+\frac{11\sqrt{6}}{10}\right)\right],\,\left[\frac{8}{5}-\frac{11\sqrt{6}}{10},f\left(\frac{8}{5}-\frac{11\sqrt{6}}{10}\right)\right]\right]$$

$$pointslist := \left[[-5,0],\,[6,0],\,\left[\frac{8}{5},\frac{17393508}{3125}\right],\,[0,4500],\,[-5,0],\,\left[\frac{8}{5}+\frac{11\sqrt{6}}{10},\left(\frac{33}{5}+\frac{11\sqrt{6}}{10}\right)^3\left(-\frac{22}{5}+\frac{11\sqrt{6}}{10}\right)^2\right],\,\left[\frac{8}{5}-\frac{11\sqrt{6}}{10},\left(\frac{33}{5}-\frac{11\sqrt{6}}{10}\right)^3\left(-\frac{22}{5}-\frac{11\sqrt{6}}{10}\right)^2\right]\right] \tag{2.12}$$

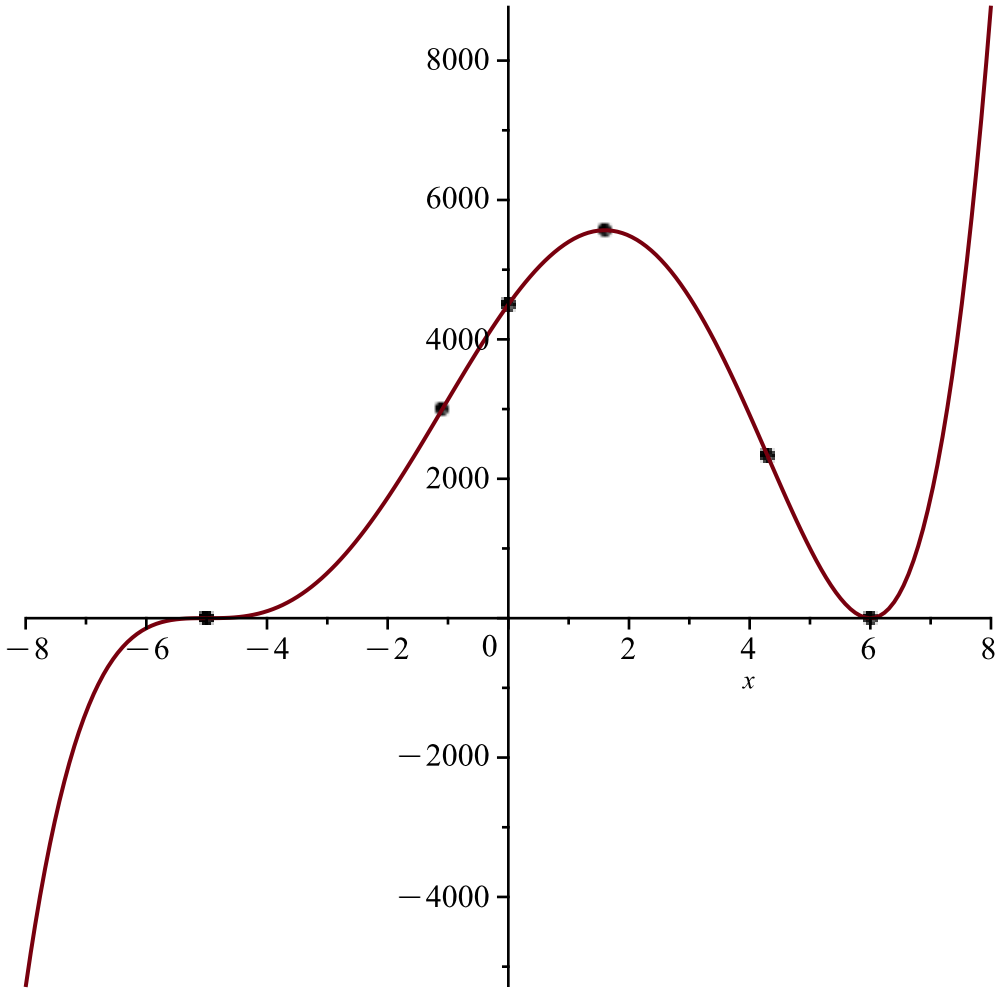
$fplot := plot(f(x), x=-8..8)$



$mypointsplotted := pointplot(pointslist, symbol = solidcircle, symbolsize = 11)$



display([*mypointsplotted*, *fplot*])

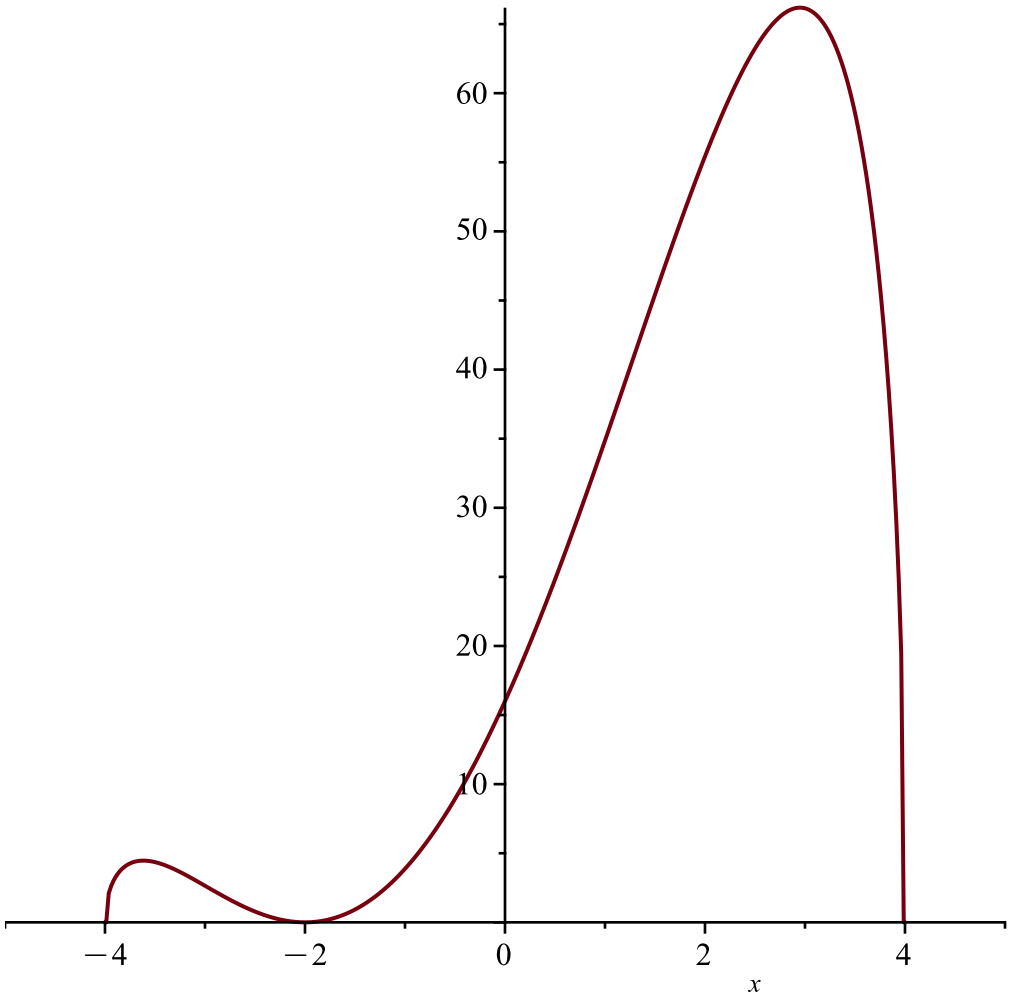


#3

$f := x \to (x+2)^2 \cdot \mathrm{sqrt}\left(16 - x^2\right)$

$$f := x \mapsto (x+2)^2 \cdot \sqrt{16 - x^2} \tag{3.1}$$

$fplot := plot(f(x), x = -5..5)$



$fp := \mathrm{D}(f)$

$$fp := x \mapsto 2\cdot(x+2)\cdot\sqrt{16-x^2} - \frac{(x+2)^2\cdot x}{\sqrt{16-x^2}} \tag{3.2}$$

$solve(fp(x) = 0)$

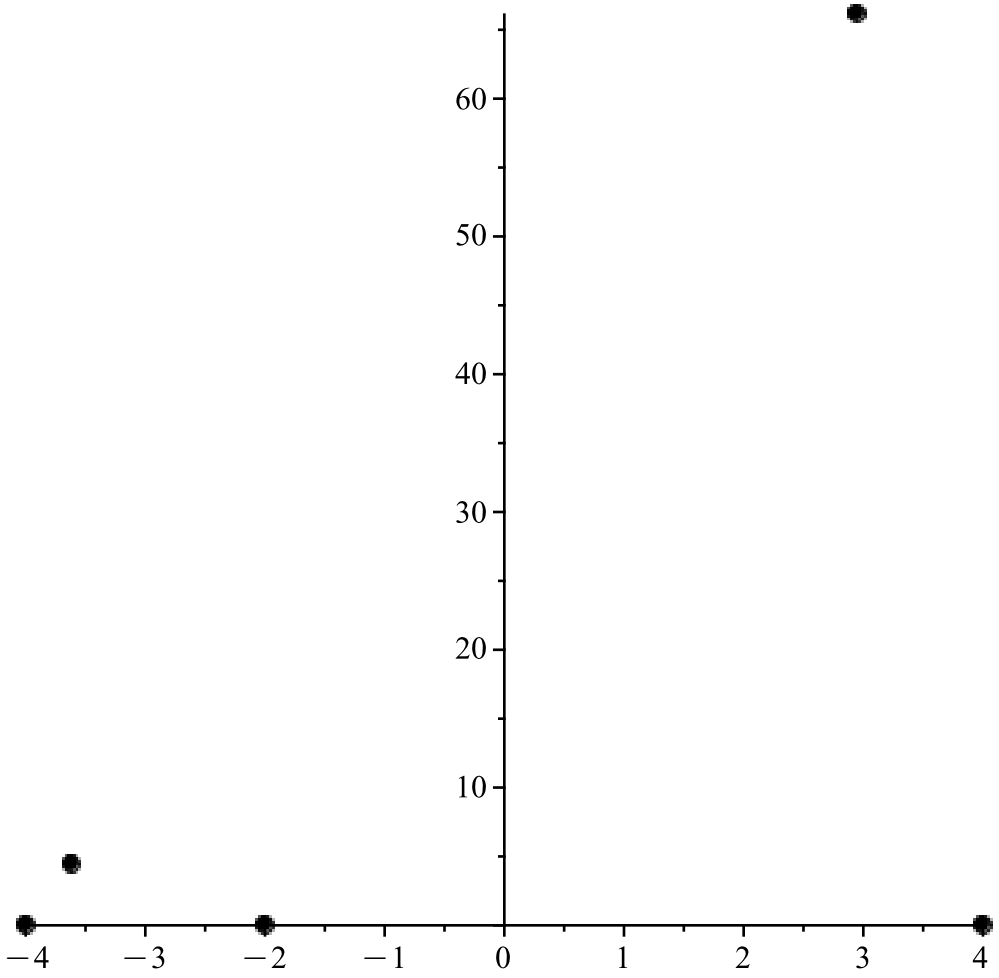
$$-2, -\frac{1}{3} - \frac{\sqrt{97}}{3}, -\frac{1}{3} + \frac{\sqrt{97}}{3} \tag{3.3}$$

$$mypoints := \left[[-4, f(-4)], [-2, f(-2)], \left[-\frac{1}{3} - \frac{\sqrt{97}}{3}, f\left(-\frac{1}{3} - \frac{\sqrt{97}}{3}\right)\right], \left[-\frac{1}{3} + \frac{\sqrt{97}}{3}, f\left(-\frac{1}{3} + \frac{\sqrt{97}}{3}\right)\right], [4, f(4)]\right]$$

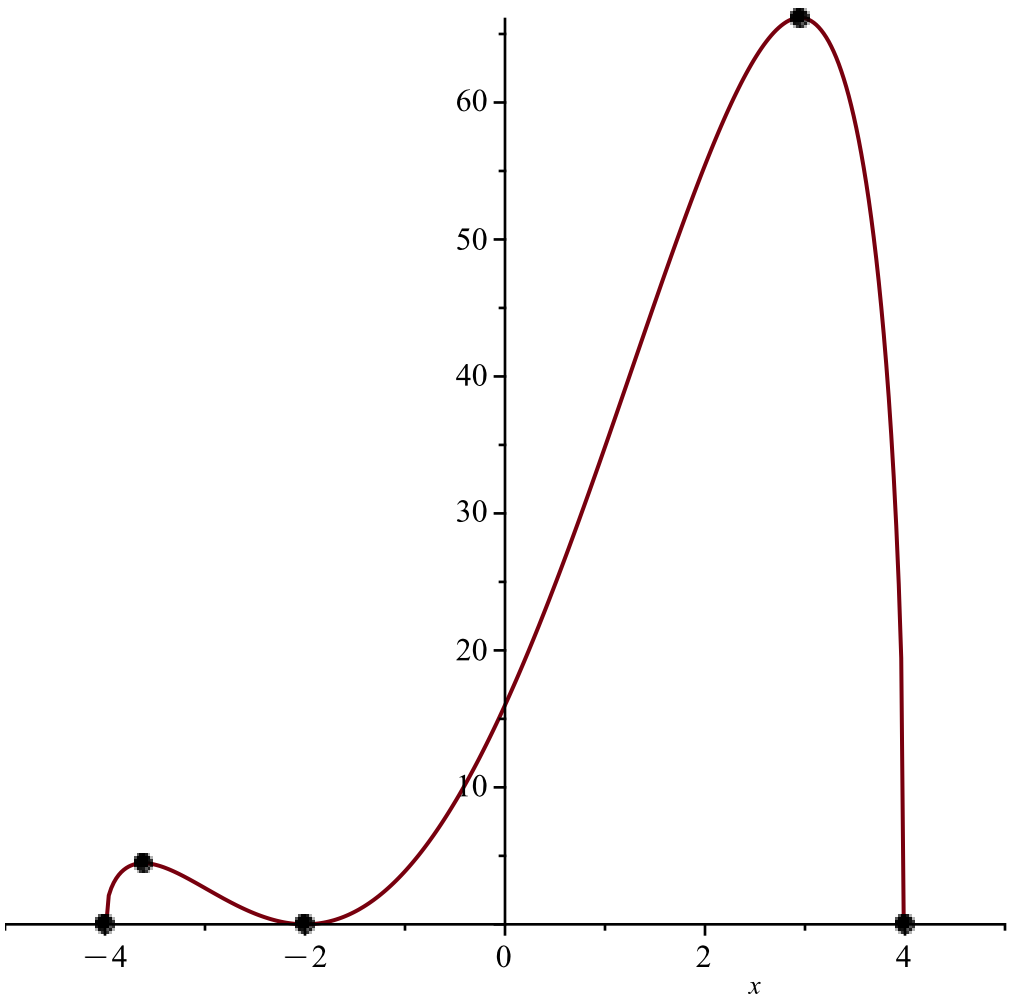
$$mypoints := \left[[-4, 0], [-2, 0], \left[-\frac{1}{3} - \frac{\sqrt{97}}{3}, \left(\frac{5}{3} - \frac{\sqrt{97}}{3}\right)^2\sqrt{-\left(-\frac{1}{3} - \frac{\sqrt{97}}{3}\right)^2 + 16}\right], \left[-\frac{1}{3} + \frac{\sqrt{97}}{3}, \left(\frac{5}{3}\right.\right.\right. \tag{3.4}$$

$$+\frac{\sqrt{97}}{3}\Bigg)^2\sqrt{-\left(-\frac{1}{3}+\frac{\sqrt{97}}{3}\right)^2+16}\Bigg],[4,0]\Bigg]$$

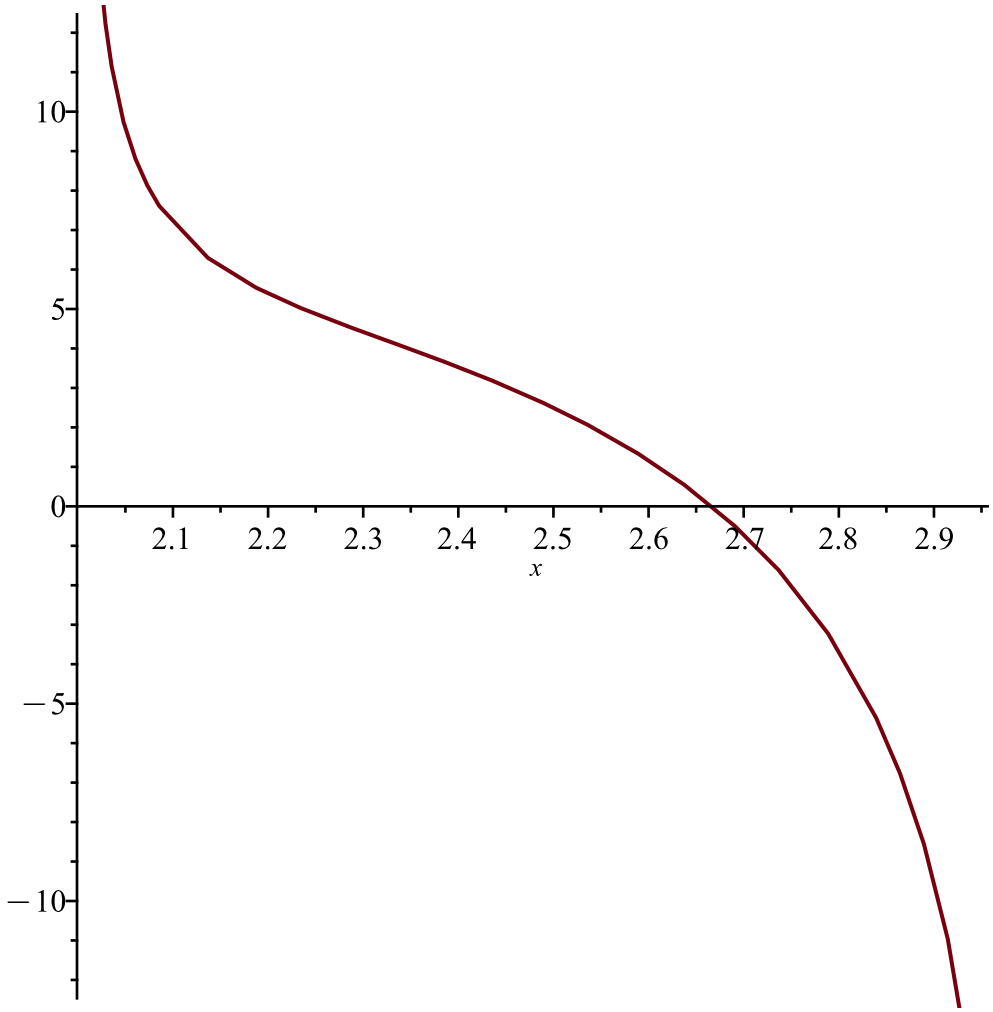
$mypointsplot := pointplot(mypoints, symbol = solidcircle, symbolsize = 12)$



$display([fplot, mypointsplot])$



$plot(fp(x), x = -5..5)$



#4

$f := x \to x \cdot (x-3)^{\frac{5}{7}}$

$$f := x \mapsto x \cdot (x-3)^{5/7} \tag{4.1}$$

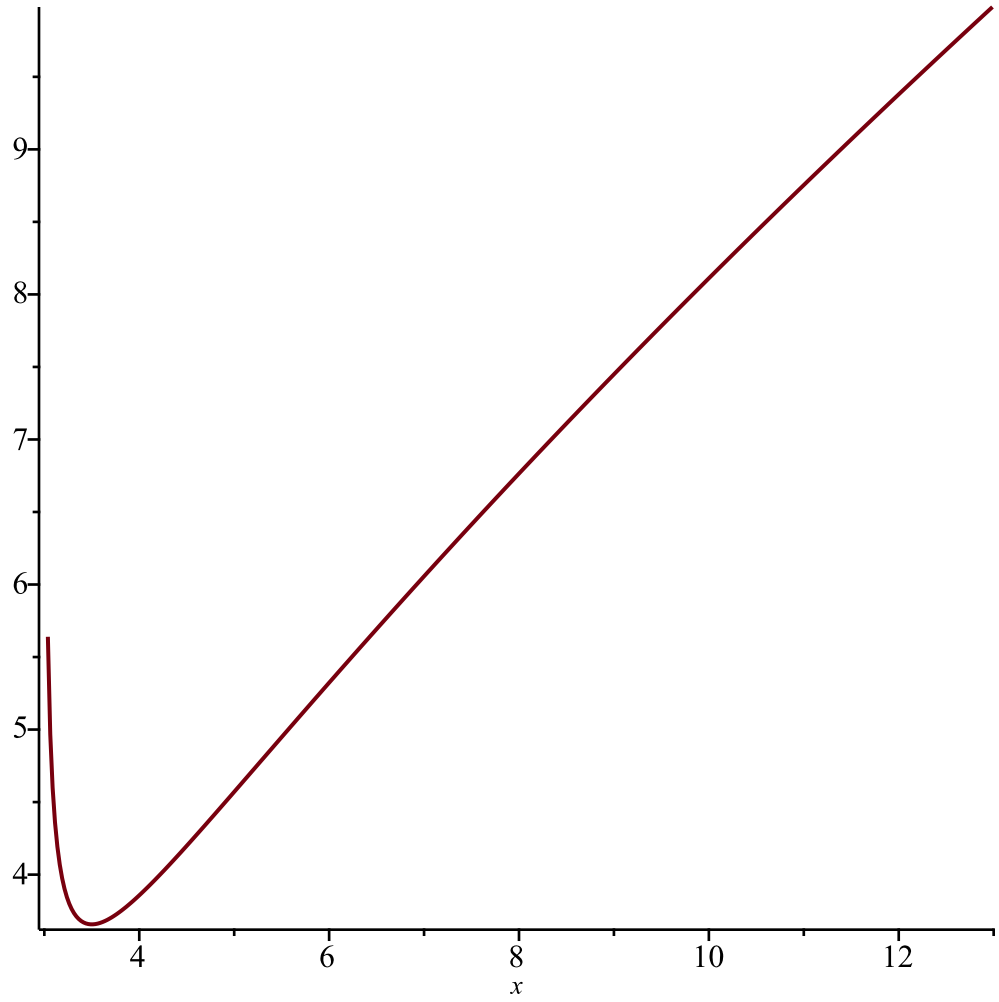
$fp := \mathrm{D}(f)$

$$fp := x \mapsto (x-3)^{5/7} + \frac{5 \cdot x}{7 \cdot (x-3)^{2/7}} \tag{4.2}$$

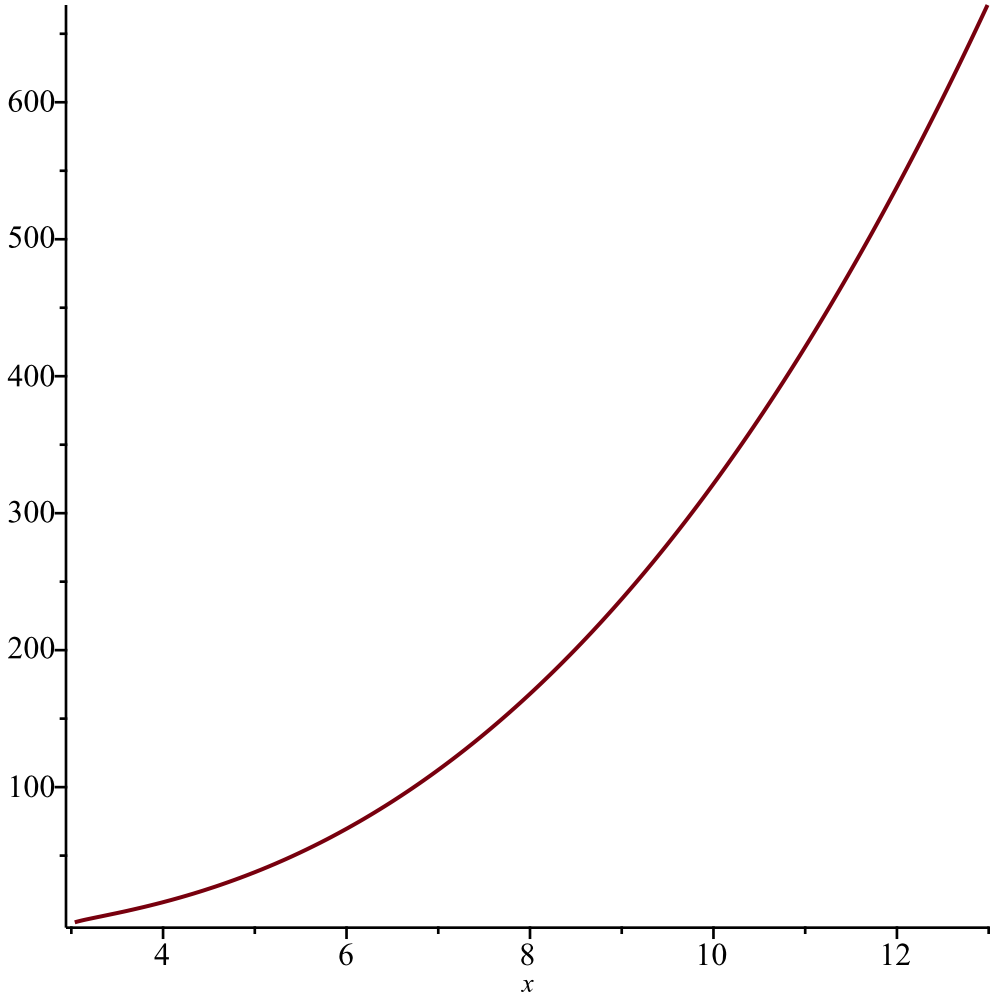
$solve(fp(x) = 0)$

$$\frac{7}{4} \tag{4.3}$$

$plot(fp(x))$



$plot(f(x))$

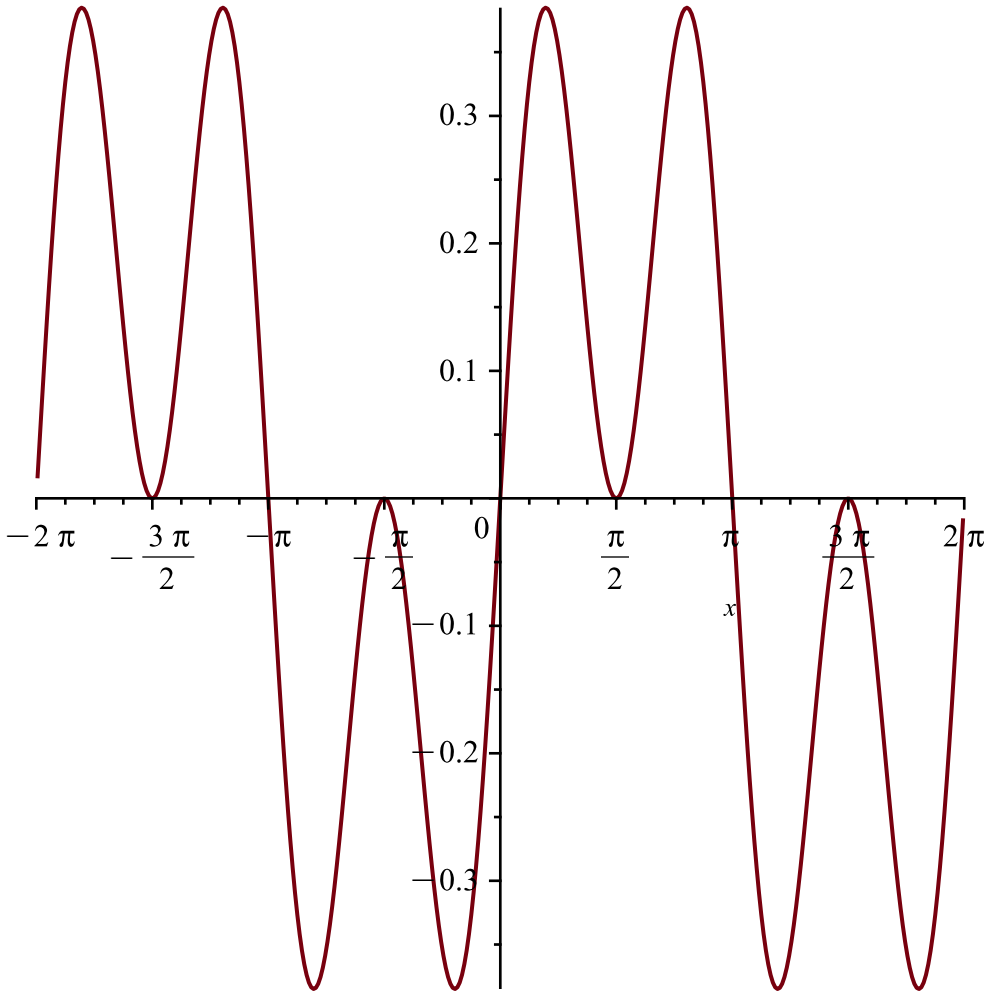


#4

$f := x \to \sin(x)$

$$f := x \mapsto \sin(x) \cdot \cos(x)^2 \qquad \textbf{(5.1)}$$

$plot(f(x))$



$fp := D(f)$

$$fp := x \mapsto \cos(x)^3 - 2\cdot\sin(x)^2\cdot\cos(x) \quad \textbf{(5.2)}$$

$solve(fp(x) = 0)$

$$\arccos\left(\frac{\sqrt{6}}{3}\right), \pi - \arccos\left(\frac{\sqrt{6}}{3}\right), \frac{\pi}{2} \quad \textbf{(5.3)}$$

fpp

$f := x \to 2x^3 - 6x^2 - 90x$

$$f := x \mapsto 2\cdot x^3 - 6\cdot x^2 - 90\cdot x \quad \textbf{(1)}$$

$fp := D(f)$

$$fp := x \mapsto 6\cdot x^2 - 12\cdot x - 90 \quad \textbf{(2)}$$

$solve(fp(x) =$