§3.2

34. At 2:00 PM a car's speedometer reads 30 mi/h. At 2:10 PM it reads 50 mi/h. Show that at some time between 2:00 and 2:10 the acceleration is exactly 120 mi/h^2.

(34) Given @ 2pm a car's speed is $30 \frac{mi}{hr}$
& @ 2:10pm " " " " $50 \frac{mi}{hr}$

We show that the acceleration is exactly $120 \frac{mi}{hr^2}$

Let s = the speed of the car in $\frac{mi}{hr}$, as a function of t = the # of hours after 2pm. Lexicon.

2pm $\Rightarrow t = 0$

2:10pm $\Rightarrow t = (10\,min)\left(\frac{1\,hr}{60\,min}\right) = \frac{1}{6}$

Then $s'(t)$ = acceleration of the car.

Let's see what $m_{AVG} = m_{SEC} = \dfrac{s(\frac{1}{6}) - s(0)}{\frac{1}{6} - 0} = \dfrac{50-30}{\frac{1}{6}} = \dfrac{20}{\frac{1}{6}} = 120 \frac{mi}{h^2}$

If we assume there's no hyperspace or quantum changes in speed, then $s'(t)$ exists on $(0, \frac{1}{6})$, $s(t)$ is cnt^s on $[0, \frac{1}{6}]$ and there is a $t \in (0, \frac{1}{6}) \ni s'(t) = m_{AVG} = 120 \frac{mi}{hr^2}$.

" $\Rightarrow$ " "implies" $A \Rightarrow B$

" $\Leftrightarrow$ " "if and only if" $A \Leftrightarrow B$ "iff"

$\ni$ "such that, so that"

$\exists$ "there is, there exists."

$\forall$ "for all, for each, for every"

$\in$ " is in " $x \in (1,2)$

$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}$
$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}$ Sets

$A \vee B$ "A OR B"
$A \wedge B$ "A AND B"

Recall $AB = 0$ means
$A = 0$ OR $B = 0$

$AB \neq 0$ means
$A \neq 0$ AND $B \neq 0$

Logic

$\text{Not}(A \vee B) = \text{NOT } A \wedge \text{NOT } B$

$\text{NOT}(A \wedge B) = \text{NOT } A \text{ OR NOT } B$

$\text{Not}(A \cup B) = \text{NOT } A \cap \text{NOT } B$

Show that $\exists\, c \in (1,2) \ni f'(c) = \dfrac{f(2)-f(1)}{2-1}$

for $f(x) = \dfrac{x-2}{x-7}$

f is cnts & difbl on its domain $= \mathbb{R} \setminus \{7\} = (-\infty,7) \cup (7,\infty)$

Property of polynomials, rational functions, & trig functions.

& $[1,2] \subset \mathcal{D}(f)$. ∴ MVT hypotheses are satisfied.

$\Longrightarrow \exists\, c \in (1,2) \ni f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

Followup: Find c $\qquad f(x) = \dfrac{x-2}{x-7}$

$$m_{sec} = \frac{f(2)-f(1)}{2-1} = \frac{\frac{2-2}{2-7} - \frac{1-2}{1-7}}{1} = 0 - \left(\frac{-1}{-6}\right) = -\frac{1}{6} = m_{AV}.$$

$$f'(x) = \frac{1(x-7)-(x-2)(1)}{(x-7)^2} = \frac{x-7-x+2}{(x-7)^2} = \boxed{\frac{-5}{(x-7)^2} = f'(x)} \overset{SET}{=} -\frac{1}{6}$$

$$\Longrightarrow -30 = -(x-7)^2 = -30$$

$$(x-7)^2 = 30$$

$$x-7 = \pm\sqrt{30}$$

$$x = 7 \pm \sqrt{30} \longrightarrow$$

$$\boxed{c = 7-\sqrt{30}}$$

Check:

$$f'(7-\sqrt{30}) = \frac{-5}{(7-\sqrt{30}-7)^2} = \frac{-5}{(-\sqrt{30})^2} = \frac{-5}{30} = -\frac{1}{6} \checkmark$$

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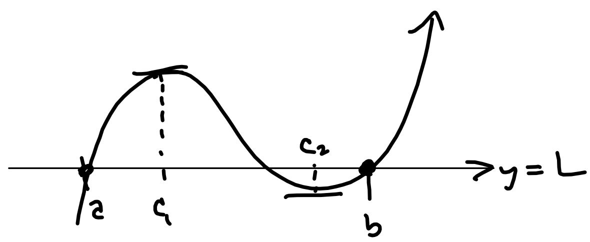
Rolle's Theorem:

If f is cnt^s on $[a,b]$ & difbl on (a,b) and if $f(a)=f(b)$, THEN $\exists\, c \in (a,b) \ni f'(c)=0$

smooth on (a,b)



Proof

$L=$

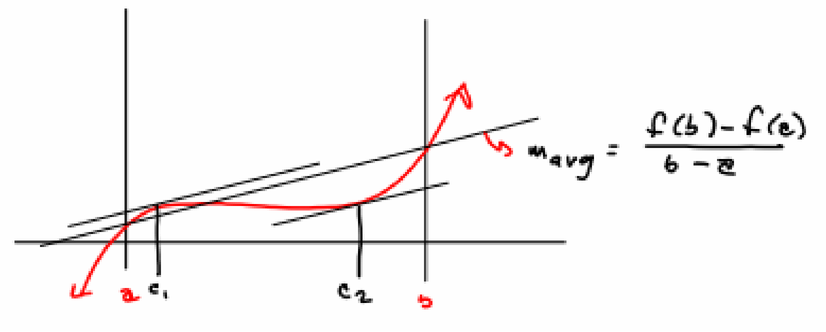
If $f(x)=f(a)=f(b)$, then $c=$ any $x \in (a,b)$ & we're done.

Assume $f(x)$ isn't constant.

By extreme value theorem, f achieves a max and a min somewhere in the interior, say $x=c$.

By Fermat's Theorem, $f'(c)=0$ ∎

Mean Value Theorem Picture:



f cnt^s on $[a,b]$ & difb^l on (a,b) $\Longrightarrow$

$\exists\, c \in (a,b) \ni f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

$\boxed{\text{Proof}}$ Define $h(x) = f(x) - \left(\dfrac{f(b)-f(a)}{b-a}(x-a) + f(a)\right)$

$y - y_1 = m(x-x_1)$

$y = m(x-x_1) + y_1$

Then $h(a) = f(a) - \left(\left(\dfrac{f(b)-f(a)}{b-a}\right)(a-a) + f(a)\right) = 0$

& $h(b) = f(b) - \left(\left(\dfrac{f(b)-f(a)}{b-a}\right)(b-a) + f(a)\right)$

$= f(b) - (f(b) \underbrace{- f(a) + f(a)})$

$= 0$

$\Longrightarrow$ $h(x)$ satisfies Rolles Theorem.

$\Longrightarrow \exists\, c \in (a,b) \ni h'(c) = 0$

i.e., $h'(c) = f'(c) - \left(\left(\dfrac{f(b)-f(a)}{b-a}\right)(1)\right) = 0$

$\Longrightarrow f'(c) = \dfrac{f(b)-f(a)}{b-a}$ ∎

It doesn't assume $f'(x)$ is cnt^s.

Just that $f'(x)$ exists.