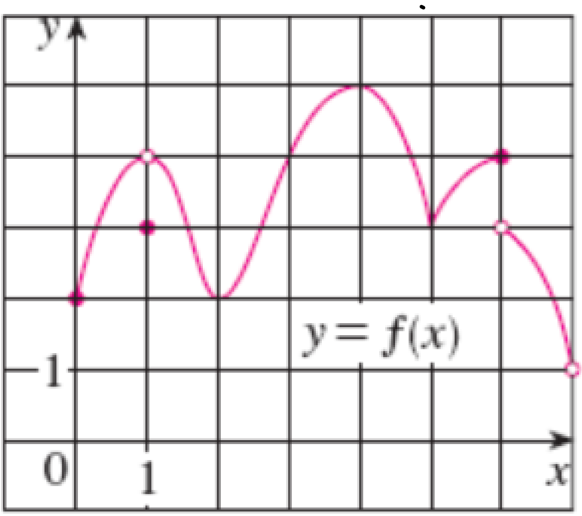
1 Definition Let c be a number in the domain D of a function f. Then $f(c)$ is the

- **absolute maximum** value of f on D if $f(c) \geqslant f(x)$ for all x in D.
- **absolute minimum** value of f on D if $f(c) \leqslant f(x)$ for all x in D.

2 Definition The number $f(c)$ is a

- **local maximum** value of f if $f(c) \geqslant f(x)$ when x is near c.
- **local minimum** value of f if $f(c) \leqslant f(x)$ when x is near c.



Abs Max : 5 WebAssign

Abs Max of $y = 5$ @ $x = 4$ Me

" " $(4,5)$

More relevant to graphing

Abs Min : (NONE!)

$R = (-1, 5]$

$(7,-1)$ is not on the graph

Local Minima : $(2,2)$, $(5,3)$, $(1,3)$, $(0,2)$ also.

Local maxima : $(4,5)$, $(6,4)$

Calculus: Local max/min when
$f'(x)=0$ OR $f'(x) \nexists$

$f(x)= x^2-3x+2$

$f'(x) = 2x-3 \overset{\text{SET}}{=} 0 \Rightarrow$

$2x=3 \Rightarrow$

$x=\frac{3}{2}$

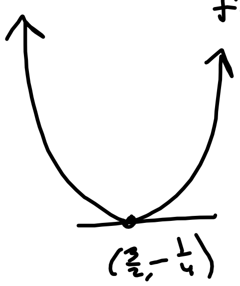
@ $\left(\frac{3}{2}, f\left(\frac{3}{2}\right)\right)$ Flat spot

$x^2-3x+2 = x^2-3x+\left(\frac{3}{2}\right)^2-\frac{9}{4}+\frac{2}{1}\cdot\frac{4}{4}$

$= \left(x-\frac{3}{2}\right)^2-\frac{1}{4}$

$(h,k) = \left(\frac{3}{2}, -\frac{1}{4}\right)$

$f(x) = a(x-h)^2+k$



$\left(\frac{3}{2}, -\frac{1}{4}\right)$

$f(x) = (x-2)^2 x = (x^2-4x+4)\,x = x^3-4x^2+4x$

$f'(x) = 3x^2-8x+4 \overset{\text{set}}{=} 0$

$3x^2-8x \quad = -4$

$3\left(x^2-\frac{8}{3}x \quad\right) = -4$

$3\left(x^2-\frac{8}{3}x+\left(\frac{4}{3}\right)^2\right) = -4 + 3\left(\frac{16}{9}\right) = -\frac{4}{1}\cdot\frac{3}{3} + \frac{16}{3} = \frac{-12+16}{3}$

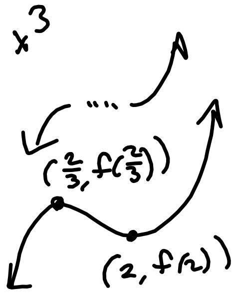
$\rightarrow 3\left(x-\frac{4}{3}\right)^2 = \frac{4}{3} \rightarrow$

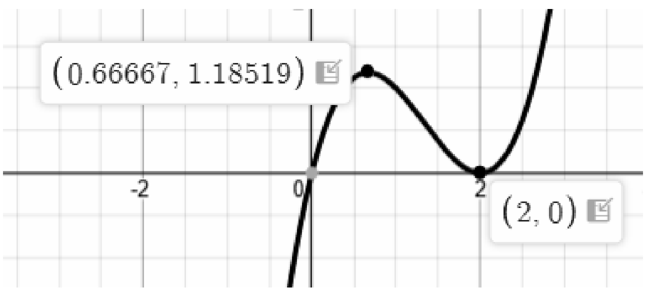
$\left(x-\frac{4}{3}\right)^2 = \frac{4}{9} \rightarrow$

$x-\frac{4}{3} = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

$x = \frac{4\pm 2}{3} = \frac{6}{3} = 2, \quad \frac{2}{3}$

x^3





Extreme Value Theorem:

If f is cnt^s on $[a,b]$, then $\exists$ an absolute max and an absolute minimum <u>in</u> that interval

Abs Max: $\exists\, c \in [a,b] \ni f(c) \geq f(x) \;\forall\, x \in [a,b]$

Abs min: " " " " " $\leq f(x)$ " " " "

Strategy:

1. Check endpoints.
2. Find all $x \ni f'(x)=0$ OR $f'(x)$ $\underbrace{\text{is undefined}}_{\nexists}$

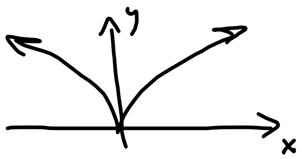
Check 'em all.

Don't be afraid to use Desmos to check, but crank out as many by hand w/ ~~graph~~ scientific calculator as you have time to do, to hone your skills for Written Tests.

Example of $f'(x) \nexists$

$f(x) = x^{\frac{2}{3}} \longrightarrow f'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$

$\nexists$ @ $x=0$.



Note $x = 0 \in \mathcal{D}(f)$, but $x = 0 \notin \mathcal{D}(f')$

CRITICAL POINTS:

$x \in \mathcal{D}(f)$ and $f'(x) = 0$ OR $f'(x) \nexists$

FINDING Extremes : Critical Points & Endpoints

FERMAT'S THEOREM If $f(x)$ is a local max (min) and $f'(x)$ $\exists$, then $f'(x) = 0$.

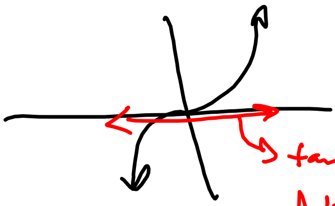
We sort of use the converse of Fermat's Theorem to find extrema.

BEWARE The converse doesn't always hold.

$f(x) = x^3$

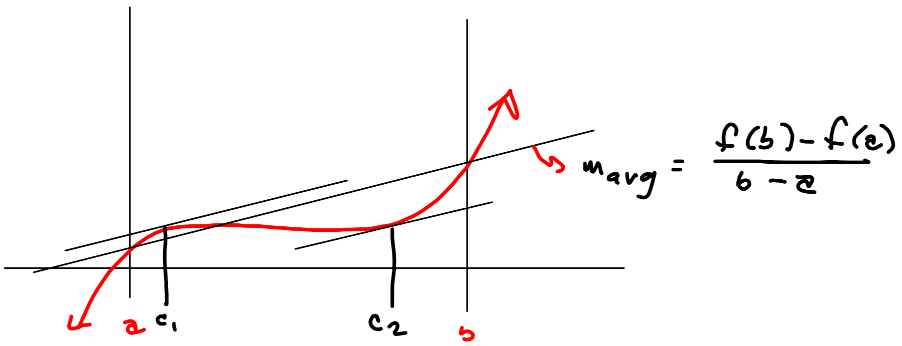
$f'(x) = 3x^2 \overset{\text{SET}}{=} 0 \longrightarrow x = 0$

$f'(0) = 0$, but $f(0)$ is a terrace point



tangent @ zero has zero slope.
NO MAX OR MIN.

Sneak Preview of 3.2:



Mean value Theorem:

f cnt^s on $[a,b]$; f difb^l on (a,b) $\longrightarrow$

$\exists\, c \in (a,b) \ni f'(c) = m_{avg} = \frac{f(b)-f(a)}{b-a}$.

See week 3 feedback.

Expect more, soon.