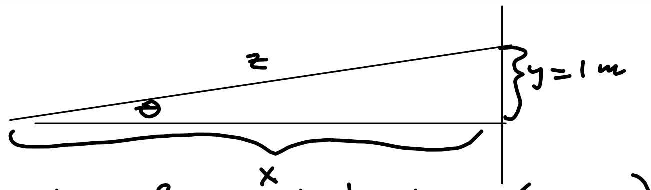
Class canceled Thursday.

This Week: 2.8, 2.9

[–/1 Points] DETAILS SCALC8 2.8.022.MI. PRACTICE ANOTHER

A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 9 m from the dock? (Round your answer to two decimal places.)



Let x = distance from boat to dock (in m) as function of t = time (in s)

Want $\left.\frac{dx}{dt}\right|_{x=9}$, given $\frac{dz}{dt} = 1\ \frac{m}{s}$

$$x^2+1^2=z^2$$

$$2xx' = 2zz'$$

$$x' = \frac{2zz'}{2x} = \frac{zz'}{x}$$

$$x=9 \Rightarrow x^2+1^2 = 9^2+1^2 = 82 = z^2$$

$$z=\sqrt{82} \longrightarrow$$

$$\left.\frac{dx}{dt}\right|_{x=9} = \frac{(\sqrt{82})(1)}{9} \approx 1.00615390424 \approx \boxed{1.01\ \frac{m}{s}}$$

§2.9 Linear Approximation and Differentials

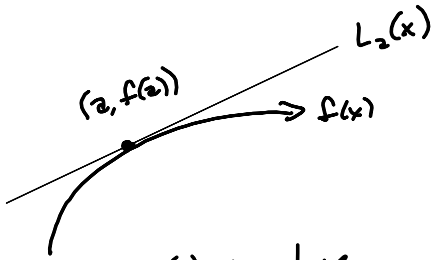
$y = m_{tan}(x - x_1) + f(x_1)$ = tangent line to $f(x)$ @ x_1

$= f'(x_1)(x - x_1) + f(x_1)$ my way

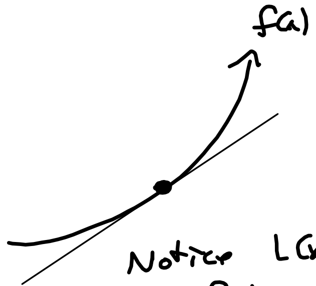
$= f(x_1) + f'(x_1)(x - x_1)$

$= f(a) + f'(a)(x - a) = L_a(x) = L(x)$ Book WAY

This is the linearization of $f(x)$ at $x = a$



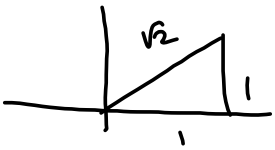
Notice $L(x)$ is above $f(x)$ near $x = a$ when f is concave down



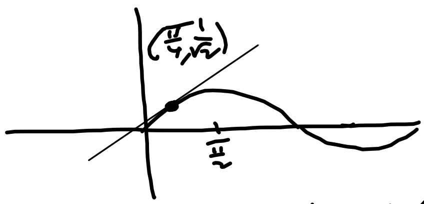
Notice $L(x)$ is below $f(x)$ when $f(x)$ is concave up.

Use a linear approximation to approximate sin(47º)

$\sin(45^\circ) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



$\sin(47^\circ)$



Tangent line to $\sin(x)$

@ $x = 45^\circ = \frac{\pi}{4}$,

FACT Must convert to radians for this stuff.

$$47^\circ = (47^\circ)\left(\frac{\pi}{180^\circ}\right)$$

$$47^\circ = 45^\circ + 2^\circ = \frac{\pi}{4} + \underbrace{\frac{2\pi}{180^\circ}}_{\Delta x \text{ = How far from } \frac{\pi}{4} \text{ we are.}} = \frac{\pi}{4} + \frac{\pi}{90}$$

$$f'(x) = \cos(x)$$

$$f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

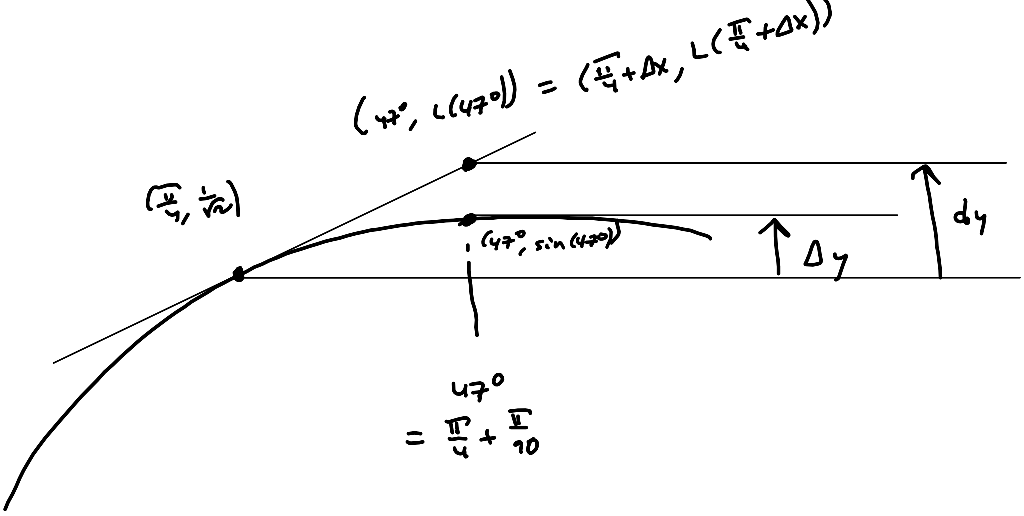
$$f'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$L_{\frac{\pi}{4}}(x) = f'(\pi/4)\left(x - \frac{\pi}{4}\right) + f\left(\frac{\pi}{4}\right) \longrightarrow$$

$$L(47^\circ) = \frac{1}{\sqrt{2}}\left(\frac{\pi}{4} + \frac{\pi}{90} - \frac{\pi}{4}\right) + \frac{1}{\sqrt{2}}$$

$$= \underbrace{\frac{1}{\sqrt{2}}\left(\frac{\pi}{90}\right)}_{dy \approx \Delta y} + \frac{1}{\sqrt{2}} \approx \frac{\pi}{90\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$dy = f'(x)dx \approx \Delta y = f(x+\Delta x) - f(x)$$



$\sin(47^\circ) \approx$ 0.731353701619 calculated

$L(47^\circ) \approx L\left(\frac{\pi}{4} + \frac{\pi}{90}\right) =$ 0.731789464176 $> \sin(47^\circ)$

~~Want to paint a~~

§2.9 # 12

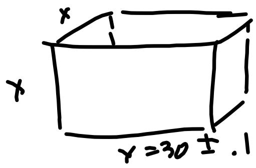
The edge of a cube was found to be 30 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum possible error, relative error, and percentage error in computing the volume of the cube and the surface area of the cube. (Round your answers to four decimal places.)

(a) the volume of the cube

maximum possible error

relative error

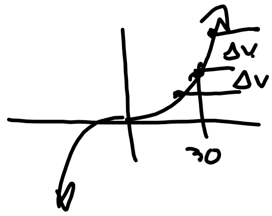
percentage error



What's the max error possible?

$V = x^3$

$\Delta V = (30.1)^3 - (30)^3$



$\approx$ 270.901 Directly.

Now using a differential

$V(x) = V = x^3 \longrightarrow$

$dV = 3x^2\,dx \approx \Delta V$

$dV\Big|_{\substack{x=30 \\ dx=\Delta x = 0.1}} = 3(30^2)(0.1) = 3(900)(.1) = 2700(.1)$

$= 270$ exactly

= Differential Approximation of the max error in volume measurement.

RELATIVE ERROR:

$$\frac{\Delta V}{V} \approx \frac{dV}{V} = \frac{270}{27000} = \frac{1}{100} = .01$$

PERCENTAGE ERROR $= \left(\frac{\Delta V}{V}\right)(100\%) = 1\%$.

Yes, Mark. Tangent Line is 1st-degree Taylor Polynomial for *f*.

Yes. This is all pointing towards Taylor Series in Calculus II.

$$f(x) \approx f(x_1) + f'(x_1)(x - x_1)$$

$$\approx f(x_1) + f'(x_1)(x-x_1) + \frac{f''(x_1)}{2}(x-x_1)^2$$

$$f(x) = f(x_1) + f'(x_1)(x-x_1) + \frac{f''(x_1)}{2}(x-x_1)^2 + \frac{f'''(x_1)}{3!}(x-x_1)^3 + \cdots$$