$x^2+2y^2=2$

$2x+4yy'=0 \longrightarrow 4yy'=-2x$

$y'=-\frac{2x}{4y}=-\frac{x}{2y}$

$2+4(y'y'+yy'')=0$

$\Rightarrow 2+4(y')^2+4yy''=0$

$1+2y'^2+2yy''=0$

$2yy''=-2y'^2-1$

$y''=\frac{-2y'^2-1}{2y}=\frac{-2\left(-\frac{x}{2y}\right)^2-1}{2y}=\frac{-\left(-\frac{x^2}{4y^2}\right)(2)-1}{2y}$

$=\frac{\frac{x^2}{2y^2}-1}{2y}=\frac{x^2-2y^2}{2y\cdot 2y^2}$

$=\frac{x^2-2y^2}{4y^3}\stackrel{?!}{=}-\frac{1}{2y^3}$

Book way

$x^2+2y^2=2$

$2x+4yy'=0$

$4yy'=-2x$

$y'=\frac{-2x}{4y}=-\frac{x}{2y}=\frac{f}{g}$

$y''=\left(\frac{f}{g}\right)'=\frac{f'g-fg'}{g^2}=\frac{-1(2y)-(-x)(2y')}{(2y)^2}=\frac{-2y+x\left(2\left(-\frac{x}{2y}\right)\right)}{4y^2}$

$=\frac{-2y-\frac{x^2}{y}}{4y^2}=\frac{-2y^2-x^2}{4y^3}=\frac{-(x^2+2y^2)}{4y^3}=\frac{-2}{4y^3}=\boxed{\frac{-1}{2y^3}=y''}$

$x^2+2y^2=2$

$2x+4yy'=0$

$y'=\frac{-2x}{4y}=-\frac{x}{2y}$

$2+4y'y'+4yy''=0$

$4yy''=-2-4y'^2$

$y''=\frac{-2-4y'^2}{4y}=\frac{-1-2y'^2}{4y}=\frac{-1-2\left(-\frac{x}{2y}\right)^2}{4y}$

$=\frac{-1-2\left(\frac{x^2}{4y^2}\right)}{4y}\cdot\frac{4y^2}{4y^2}=\frac{-4y^2-2x^2}{(4y)(4y^2)}$

$=\frac{-2(2y^2+x^2)}{16y^3}=\frac{-(x^2+2y^2)}{8y^3}=-\frac{2}{8y^3}=-\frac{1}{4y^3}$ ✓

$=$