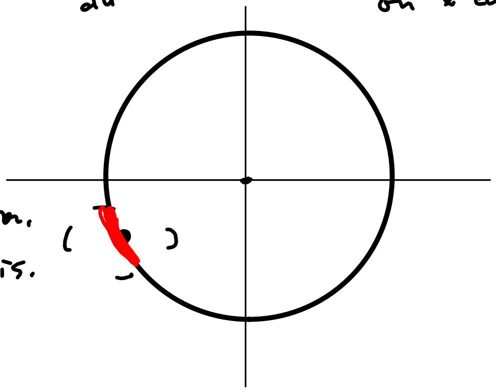
Section 2.5 - Chain Rule. Derivative of a function of a function.

$$f(x) = \left(\sin(3x^2\cos(x))\right)^{73} \longrightarrow$$

$$f'(x) = \underbrace{73\left(\sin(3x^2\cos(x))\right)^{72}}_{\frac{du^{73}}{du} = 73u^{72}} \underbrace{\cos(3x^2\cos(x))}_{\frac{d\sin(u)}{du} = \cos(u)} \underbrace{\left(6x\cos(x) + 3x^2(-\sin(x))\right)}_{\text{Product Rule on } x^2\cos(x)}$$

Section 2.6



$x^2+y^2=25$

is not a function.

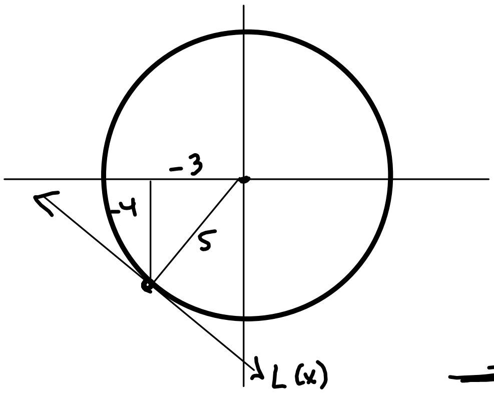
But locally, it is.

OLD WAY

Solve for y:

$y^2 = 25-x^2$

$\longrightarrow y = \pm\sqrt{25-x^2}$ $\longrightarrow$ $y=\sqrt{25-x^2}$; $y=-\sqrt{25-x^2}$



Find an equation of the tangent line to this circle at the point $(-3,-4) = (x_1,y_1) = (x_1, f(x_1))$,

where

$f(x) = -\sqrt{25-x^2} = -(25-x^2)^{\frac{1}{2}}$

$\longrightarrow f'(x) = -\frac{1}{2}(25-x^2)^{-\frac{1}{2}}(-2x)$

$L(x)$ = Linearization of $f(x)$

= Tangent Line for $f(x)$

$= f'(x_1)(x-x_1) + f(x_1)$

$= f'(x_1)(x - x_1) + f(x_1)$

$= f'(-3)(x-(-3)) + f(-3)$

Scratch

$f'(-3) = -\frac{1}{2}(25-(-3)^2)^{-\frac{1}{2}}(-2(-3))$

$= -\frac{1}{2}(25-9)^{-\frac{1}{2}}(6)$

$= -3(16)^{-\frac{1}{2}} = -(16^{\frac{1}{2}})^{-1} = -3(4)^{-1} = -3(\frac{1}{4}) = -\frac{3}{4}$

$f(-3) = -4$, by problem build

$= \boxed{-\frac{3}{4}(x+3) - 4 = L(x)}$

Notice that at ANY point on the circle (except for (-5,0) and (5, 0)), it's locally a function. We assume that the equation implicitly defines *y* as a function of *x*.

Chain Rule

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

$$\frac{d}{dx}\left[(g(x))^2\right] = 2g(x)g'(x)$$

$$\frac{d}{dx}\left[g^2\right] = 2gg'$$ Book would say $\frac{dg}{dx}$

$$\frac{d}{dx}\left[\sin(y)\right] = \cos(y)\,y'$$

$$\frac{d}{dx}\left[\underset{f}{x^2}\,\underset{g}{y}\right] = \overset{f'}{2x}\,\overset{g}{y} + \overset{f}{x^2}\,\overset{g'}{y'}$$

$$\frac{d}{dx}\left[y\right] = \frac{dy}{dx} = y'$$

$$\frac{d}{dx}\left[x^2y^5\right] = 2xy^5 + x^2(5y^4)y'$$

$x^2+y^2=25$ ⟶

$2x+2yy'=0$ ⟶

$2yy'=-2x$

$y'=\frac{-2x}{2y} = -\frac{x}{y}$ ⟶

$$y'\Big|_{(-3,-4)} = -\frac{-3}{-4} = -\frac{3}{4} = m_{tan}$$

$L(x) = m_{tan}(x-x_1)+y_1$

$= \boxed{\frac{3}{4}(x+3)-4 = L(x)}$

Find $\frac{dy}{dx}$ for

$$\frac{x^2}{x+y} = y^2+4$$

$$\frac{(2x)(x+y) - x^2(1+y')}{(x+y)^2} = 2yy'$$

$\Longrightarrow$ $2x^2+2xy - x^2 - x^2y' = (2yy')(x+y)^2 = (2yy')(x^2+2xy+y^2)$

$\Longrightarrow$ $-x^2y' - 2y(x^2+2xy+y^2)y' = -2x^2-2xy+x^2 = -x^2-2xy$

$\Longrightarrow$ $(-x^2-2y(x^2+2xy+y^2))y' = -x^2-2xy$

$$\boxed{y' = \frac{-x^2-2xy}{-x^2-2y(x^2+2xy+y^2)}}$$

$$= \frac{x^2+2xy}{x^2+2y(x^2+2xy+y^2)}$$

2.6 #17. Think similar triangles.

$$x^2y^2 - 2xy^3 = \cos(xy) \longrightarrow$$

$$2xy^2 - x^2(2yy') - 2y^3 - 2x(3y^2y') = (-\sin(xy))(y + xy')$$

$+$

$$= -y\sin(xy) - x\sin(xy)\,y'$$

$$-2x^2yy' - 2x(3y^2)y' + x\sin(xy)\,y' = -y\sin(xy) - 2xy^2 + 2y^3$$

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$$\longrightarrow \boxed{y' = \frac{-y\sin(xy) - 2xy^2 + 2y^3}{-2x^2y - 6xy^2 + x\sin(xy)}}$$

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