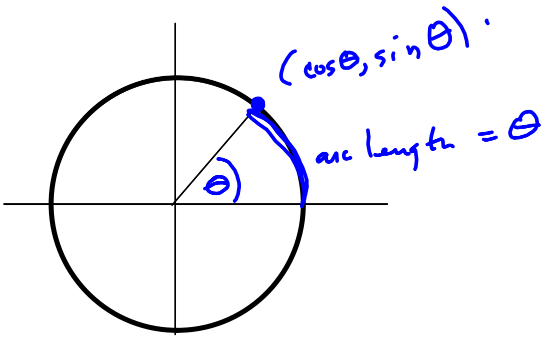
Today: Sneak Preview of Chain Rule.

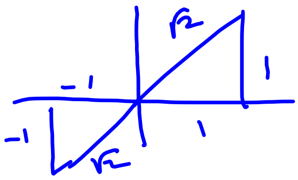
You may work on 2.3 and 2.4, today, but I'm giving those who are plowing ahead the opportunity to see a quick, easy take on the Chain Rule.

Chain Rule: The derivative with respect to x of a function of another function.

The trick is to see the function living inside the other function.

$\tan\theta = 1$





$1-1-\sqrt{2}$

$1-2-\sqrt{3}$

$0-1-1$

$3-4-5$

Suppose that *f* is the number of miles I can drive as a function of the amount of fuel *x* in the tank.

Suppose $f(x) = x^2 + 5x$

Suppose there's a brand-new super-fuel that acts like the square of the old kind of fuel.

Now $f(x)$ looks like $f(x^2)$

Now we have the super-fuel miles function

$h(x) = f(x^2) = (x^2)^2 + 5x^2 = x^4 + 5x^2$

= how far I can go, now, on *x* amount of fuel!

Think of $h(x)$ as $h(x) = f(x^2) = f(g(x))$ where

$g(x) = x^2$

Then $h'(x) = 4x^3 + 10x^2$ → No $4x^3 + 10x$ Sheesh!

Check this out:

$f(x) = x^2 + 5x$

$f(x^2) = (x^2)^2 + 5(x^2)$

$g(x) = x^2$

$g'(x) = 2x$

$\frac{df}{dx} =$

$f(x) = x^2 + 5x$

$g(x) = x^2$

$f(g) = g^2 + 5g$

$\frac{df}{dg} = 2g + 5$

$\frac{dg}{dx} = 2x$

$\frac{df}{dg} \cdot \frac{dg}{dx} = (2g+5)(2x)$

$= (2x^2+5)(2x) = 4x^3 + 10x$

$= f'(g(x))\, g'(x)$

$$\frac{d}{dx}\left[(x^2+5)^2\right] = \frac{d}{dx}\left[x^4+10x^2+25\right] = 4x^2+20x$$

$$\left.\begin{aligned} g(x) &= x^2+5 \\ f(x) &= x^2 \end{aligned}\right\} h(x) = f(g(x))$$

$$\Rightarrow \frac{dh}{dx} = \frac{df}{dg}\cdot\frac{dg}{dx} = \frac{d}{dg}\left[g^2\right]\frac{d}{dx}\left[g\right]$$

$$= (2g)(2x)$$

$$= 2(x^2+5)(2x)$$

$$= 4x^2+20x$$

$$\frac{d}{dx}\left[\sin^2 x\right]$$

$$2\sin(x)\cdot\cos(x)$$

$$\frac{d}{dx}\left[(x^2+5x+2)^{23}\right]$$

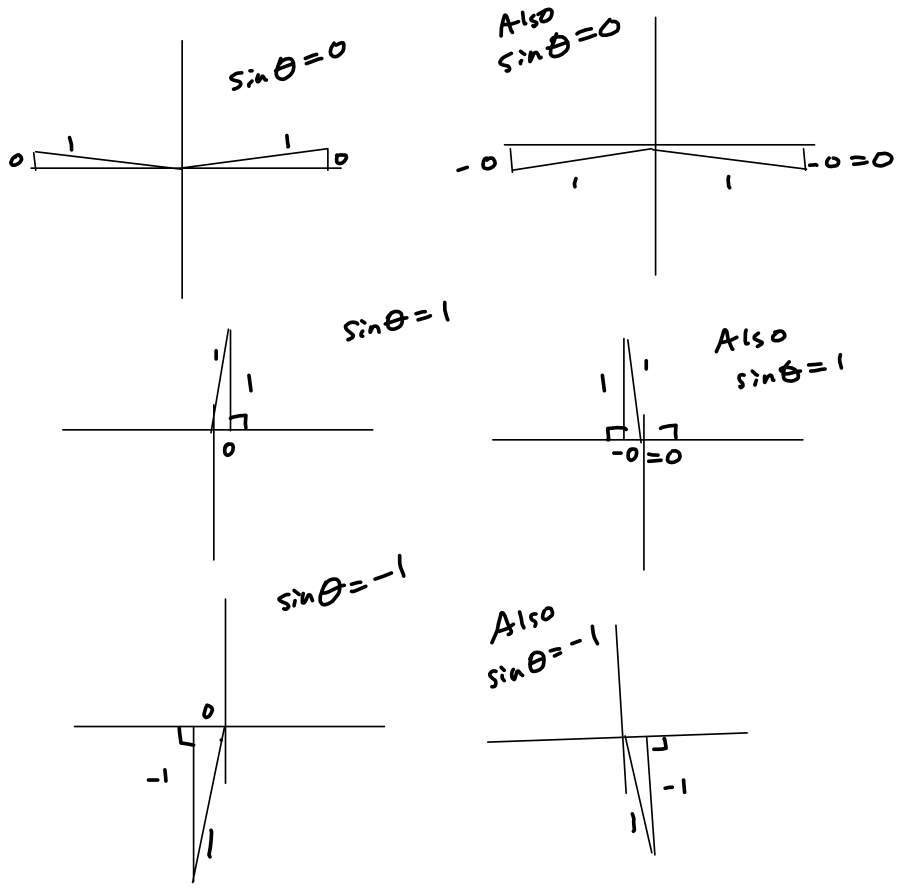
$$= 23(x^2+5x+)^{22}(2x+5)$$

$$\frac{d}{dx}\Big[\underbrace{(x^2-5)^5}_{f}\underbrace{(\sin^2(x)+\tan(x))^{37}}_{g}\Big]$$

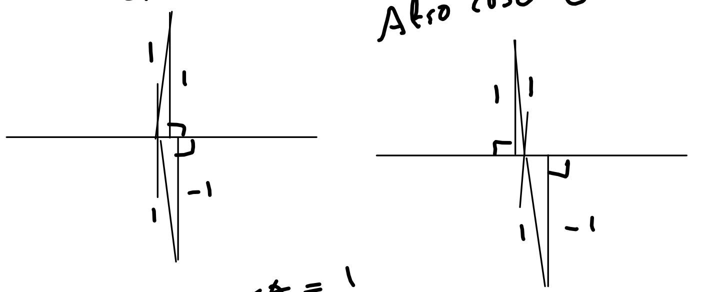
$$= \underbrace{5(x^2-5)^4(2x)}_{f'}\underbrace{(\sin^2(x)+\tan(x))^{37}}_{g} + \underbrace{(x^2-5)^5}_{f}\cdot\underbrace{37(\sin^2 x+\tan x)^{36}(2\sin(x)\cos(x)+\sec^2(x))}_{g'}$$

The above is a final answer for me and *also* for WebAssign!

Moar Degenerate Triangles

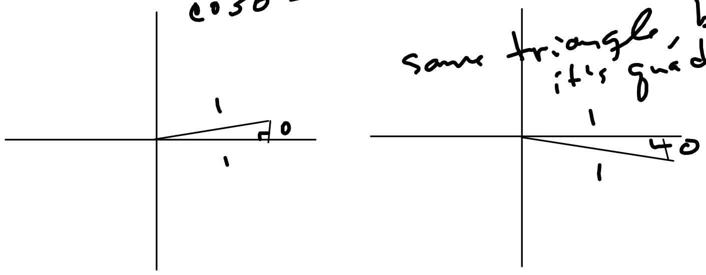


$\cos\theta = 0$ Also $\cos\theta = 0$



$\cos\theta = 1$

same triangle, because it's quadrant angle.



Intuitive idea of Chain Rule.

Let f and g be differentiable functions on their respective domains. Assume that the domain of f has some overlap with the range of g. (f can eat what g spits out)

Assume also that f and g are both differentiable on some nontrivial domain.

So, basically, assume that it makes sense to talk about the derivative of the composition

$f \circ g$.

This is by no means a formal proof, but we sort of justify it by formal symbol manipulation, treating these dh's, dg's, and dx's like they're numbers and obey the rules of arithmetic, namely, commutativity of multiplication:

$h(x) = (f \circ g)(x) = f(g(x))$

$$\frac{dh}{dx} = \frac{df(g(x))}{dx} = \frac{d\,f(g)}{dx} = \frac{d\,f(g)}{dx} \cdot \frac{d(g)}{d(g)} = \frac{d\,f(g)}{dg} \cdot \frac{dg}{dx}$$

$$= f'(g)g'(x) = \underbrace{f'(g(x))}_{\text{Here, the derivative is with respect to the "variable" } g(x).} g'(x).$$