Recall:

$(f \pm g)' = f' \pm g'$ respects sums

$(af)' = af'$ for constant .. scalar multples

$0' = 0$

$(fg)' = f'g + fg'$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$

Next up §2.4 Trig Derivatives.

The theory for this is covered in the Homework Notes and Videos on harryzaims.com.

Typical 2.4 ideas and questions.

Differentiate:

$$y = \frac{\sin(x)}{\tan(x)+1}$$

Facts:

$$\frac{d}{dx}[\sin(x)] = \cos(x)$$

$$\frac{d}{dx}[\cos(x)] = -\sin(x)$$

$$\frac{d}{dx}[\tan(x)] = \sec^2(x)$$

$$\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$$

$$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$$

$$\frac{d}{dx}[\cot(x)] = -\csc^2(x)$$

We prove these 2 in homework notes & videos. Then use them to prove the rest.

$$\frac{d}{dx}[\tan(x)] = \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right] = \left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$$

$$= \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)} = \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

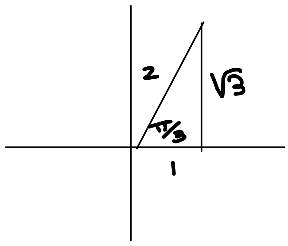
$$= \sec^2(x) \blacksquare$$

Suppose $f(\pi/3) = 4$ and $f\,'(\pi/3) = -7$, and let $g(x) = f(x)\sin(x)$ and $h(x) = \cos(x)/f(x)$. Find the following.

(a) $g'(\pi/3)$

$$g(x) = f(x)\sin(x) \implies$$

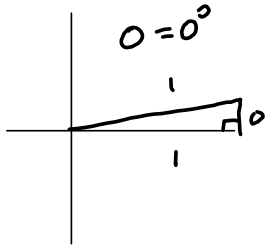
$$g'(x) = f'(x)\sin(x) + f(x)\cos(x) \implies$$

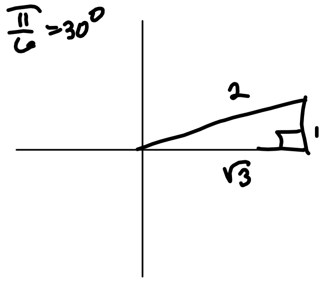


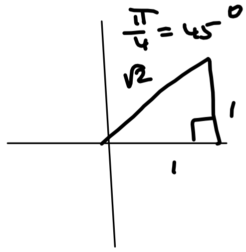
$$g'\left(\tfrac{\pi}{3}\right) = f'\left(\tfrac{\pi}{3}\right)\sin\left(\tfrac{\pi}{3}\right) + f\left(\tfrac{\pi}{3}\right)\cos\left(\tfrac{\pi}{3}\right)$$

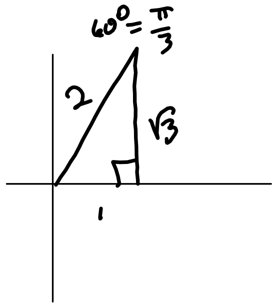
$$= (-7)\left(\tfrac{\sqrt{3}}{2}\right) + (4)\left(\tfrac{1}{2}\right)$$

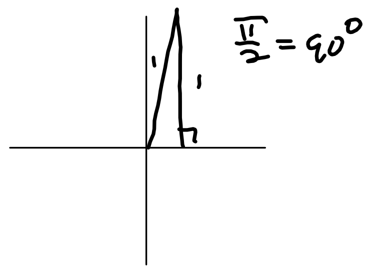
$$= \boxed{\tfrac{7\sqrt{3}}{2} + 2} = g'\left(\tfrac{\pi}{3}\right)$$



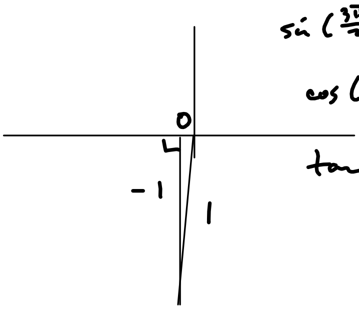








$\sin\left(\frac{3\pi}{2}\right)$



$\sin\left(\frac{3\pi}{2}\right) = \frac{-1}{1}$

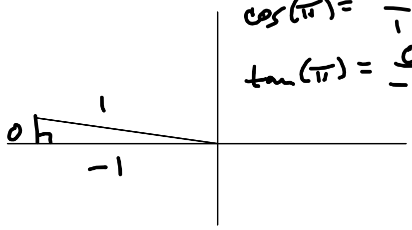
$\cos\left(\frac{3\pi}{2}\right) = \frac{0}{1}$

$\tan\left(\frac{3\pi}{2}\right) = \frac{-1}{0}\ \nexists$

$\sin(\pi) = \frac{0}{1} = 0$

$\cos(\pi) = \frac{-1}{1} = -1$

$\tan(\pi) = \frac{0}{-1} = 0$



Coming up:

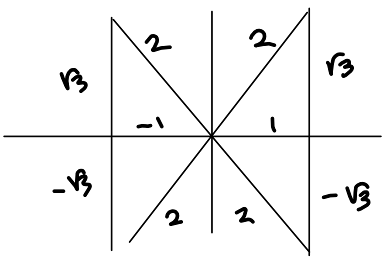
Solve

$4\sin^2\theta - 3 = 0$

$4\sin^2\theta = 3$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$



$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ captures all of 'em in $(0, 2\pi)$

All of them:

$\frac{\pi}{3} + 2n\pi$

$\frac{2\pi}{3} + 2n\pi$

$\frac{4\pi}{3} + 2n\pi$

$\frac{5\pi}{3} + 2n\pi$

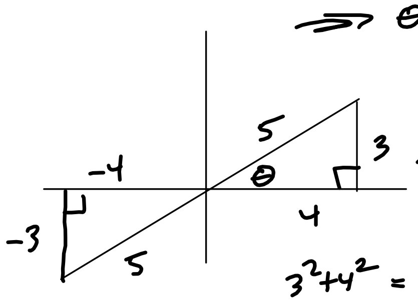
$\rightarrow \frac{\pi}{3} + n\pi$

$\frac{2\pi}{3} + n\pi$

$\tan\theta = \frac{3}{4}$

$\Rightarrow \theta = \arctan\left(\frac{3}{4}\right),\ \pi + \arctan\left(\frac{3}{4}\right)$ RADIANS

OR $\arctan\left(\frac{3}{4}\right),\ \underbrace{180^\circ + \arctan\left(\frac{3}{4}\right)}$



37° ish.

Reference angle is $\arctan\left(\frac{3}{4}\right)$
Add that to $\pi = 180^\circ$

$3^2 + 4^2 = 9 + 16 = 25$

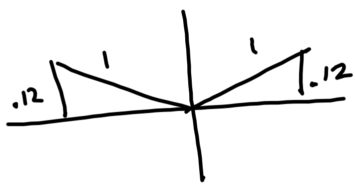
$\sqrt{25} = 5 = r$

53° ish

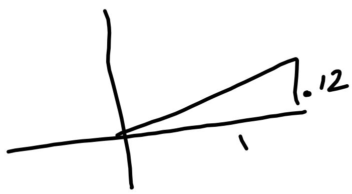
53.1301023542

$\arctan\left(\frac{3}{4}\right) \approx 36.8698976458^\circ$

$\sin\theta = .12$



$\tan\theta = .12$



Slurp up the Notes and Videos on harryzaims.com.