Looking at Score View on WebAssign

We're midweek and most of you haven't even started Section 2.1

One of you has. Good work!

These deadlines are "last dyin' dog" deadlines. You want to be ahead and push ahead all the time, instead of passively waiting for lecture and then starting the work.

Get ahead and stay ahead. I should have been answering questions last time on 2.1 and/or 2.2, yesterday, but apparently, almost no one has even looked at the new chapter, yet. This is a recipe for low grades.

Take a bite out of the material, early. Put it down when stuck and come to class loaded with questions.

I don't want to lecture to empty vessels. I want to engage with students who are mature enough to engage with the material on their own, at least a little, and as you continue in that way, a lot...

Be an ACTIVE learner, and it's easier. Be a PASSIVE learner, and always be rushed. Always be behind the 8-ball, and wondering why.

§2.3

Power Rule for $f'(x)$ ← Liebniz Notation

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}$$

$$\lim_{x_2 \to x} \frac{f(x_2) - f(x_1)}{x_2 - x_1} \approx \frac{y_2 - y_1}{x_2 - x}$$

$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$ Power Rule $(n \neq 0)$

$\frac{d}{dx}\left[x^5\right] = 5x^4$

$\frac{d}{dx}\left[c\right] = 0$ Horizontal Line $f(x) = y = c =$ constant

$\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right]$

$\frac{d}{dx}\left[cf(x)\right] = c\left(\frac{d}{dx}\left[f(x)\right]\right)$

"$\frac{d}{dx}$" is a Linear Operator.

Restate the above, with Newton's "prime" notation:

$(f+g)' = f' + g'$

$(cf)' = cf'$

Differential Operator is <u>linear</u>.

Find eq'n of tangent line to $f(x) = x^2+5x+2$
@ $x=-2$. Then sketch $f(x)$ & $L(x)=$ tangent line.

§2.1/2.2 Version

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$:

$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+5(x+h)+2-(x^2+5x+2)}{h}$

$= \frac{\cancel{x^2}+2xh+h^2 \cancel{+5x}+5h\cancel{+2}\cancel{-x^2}\cancel{-5x}\cancel{-2}}{h}$

$= \frac{2xh+h^2+5h}{h} = \frac{\cancel{h}(2x+h+5)}{\cancel{h}}$

$= 2x+h+5 \xrightarrow{h\to 0} \boxed{2x+5 = f'(x)}$
$h \neq 0$

$f(-2) = (-2)^2+5(-2)+2 = 4-10+2 = -4 \rightarrow \boxed{(x_1, f(x_1)) = (-2,-4)}$

$f'(-2) = m_{tan} = 2(-2)+5 = \boxed{1 = m_{tan}}$

$f(x) = x^2+5x+2 \rightarrow$

$f'(x) = 2x+5 \rightarrow f'(-2)$
$= 2(-2)+5 = 1$
$m_{tan} = f'(-2)$

$\boxed{f(-2) = -4}$

$$\begin{array}{r|rrr} -2 & 1 & 5 & 2 \\ & & -2 & -6 \\ \hline & 1 & 3 & -4 \end{array}$$

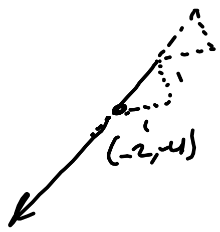
$(x_1, y_1) = (-2,-4)$

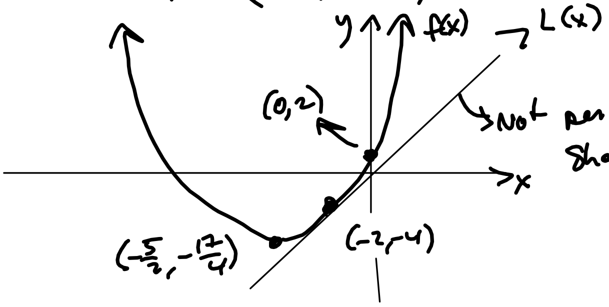
Ask About Pascal's Triangle & Binomial Theorem.

$$y = L(x) = m(x - x_1) + y_1$$
$$= f'(x_1)(x - x_1) + f(x_1)$$
$$= f'(-2)(x - (-2)) + (-4)$$
$$= \boxed{1(x+2) - 4 = L(x)}$$

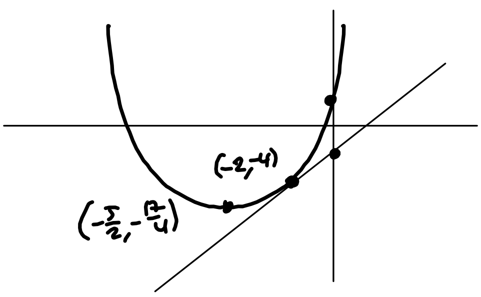
$$y = m(x - x_1) + y_1$$
$$\boxed{L(x) = 1(x - (-2)) - 4}$$

$$x^2 + 5x + 2$$
$$= \underline{x^2 + 5x + \left(\tfrac{5}{2}\right)^2} - \tfrac{25}{4} + 2\left(\tfrac{4}{4}\right)$$
$$= \left(x + \tfrac{5}{2}\right)^2 - \tfrac{17}{4}$$
$$(h, k) = \left(-\tfrac{5}{2}, -\tfrac{17}{4}\right)$$





Not perfect. Should have y-intercept of BELOW x-axis.



Better graph, qualitatively. Note the order in which I did things, deciding where the axes went after I had the essential SHAPE & key points labeled.
It goes quickly, even if you need a 2nd draft of the graph.

NO TICK MARKS
JUST LABEL
KEY POINTS
Get them relatively correct w.r.t. each other.

Do it for $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$ @ $(8,2)$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} \Rightarrow f'(8) = \frac{1}{2}(8)^{-\frac{2}{3}} = \frac{1}{2}\left(\frac{1}{(8^2)^{\frac{1}{3}}}\right)$$

$$\frac{1}{3} - 1 = \frac{1}{3} - \frac{3}{3} = -\frac{2}{3} \qquad = \frac{1}{2}\left(\frac{1}{(8^{\frac{1}{3}})^2}\right) = \frac{1}{2}\left(\frac{1}{(2)^2}\right)$$

$$= \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8} = m_{tan}$$

$$\boxed{L(x) = y = \frac{1}{8}(x-8)+2}$$

GOOD BONUS QUESTION for MIDTERM & FINAL

Find $f'(x)$ by the limit definition for

... $f(x) = \sqrt[3]{x}$

$x^3 - y^3 = \boxed{(x-y)}(x^2+xy+y^2)$ $\qquad \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$ method:

$x^3+y^3 = (x+y)(x^2-xy+y^2)$

$$\frac{f(x)-f(c)}{x-c} = \left(\frac{\boxed{\sqrt[3]{x} - \sqrt[3]{c}}}{x-c}\right)\left(\frac{\sqrt[3]{x}^2 + \sqrt[3]{xc} + \sqrt[3]{c}^2}{\sqrt[3]{x}^2 + \sqrt[3]{xc} + \sqrt[3]{c}^2}\right)$$

$$\frac{\cancel{(x-c)}}{\cancel{(x-c)}\left(\sqrt[3]{x^2} + \sqrt[3]{xc} + \sqrt[3]{c^2}\right)} \qquad \left(= \frac{1}{\sqrt[3]{x^2} + \sqrt[3]{xc} + \sqrt[3]{c^2}} \quad (x \neq c) \right)$$

optional

$$\xrightarrow{x \to c} \frac{1}{\sqrt[3]{c^2} + \sqrt[3]{c^2} + \sqrt[3]{c^2}}$$

$$= \frac{1}{3\sqrt[3]{c^2}} = \frac{1}{3c^{2/3}} = \frac{1}{3}c^{-2/3} = f'(c)$$

<u>For Diff of Cubes</u>

Find $\frac{d}{dx}[x^3]$:

$$\frac{f(x)-f(c)}{x-c} = \frac{x^3-c^3}{x-c} = \frac{\cancel{(x-c)}(x^2+xc+c^2)}{\cancel{(x-c)}} = x^2+xc+c^2$$

$x \neq c$

$$\xrightarrow{x \to c} c^2+c^2+c^2 = 3c^2 = f'(c)$$

$$\frac{d}{dx}[x^3] = 3x^2 = f'(x)$$