**Week 4 Written Is Posted**

**Let me know if this mix of informal talks and open work period are working for you.**

**Again, I have so much on video, I'd rather you hit that when you need it, rather than waiting for me to give a general talk that will never cover everything perfectly.**

**The videos have hours of content that I don't want you to necessarily sit through, twice, ready or not.**

$f(x) = \sin\left(\frac{\pi}{x}\right)$

Why doesn't $\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right)$ $\exists$?

Recall $\lim_{x \to c} f(x) = L$ means

Given any $\varepsilon > 0$, $\exists\ \delta > 0 \ni 0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$

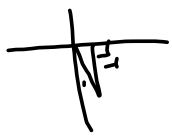
This means taking $x$ close to $c$ forces $f(x)$ close to $L$.

NOTE $\sin\left(\frac{\pi}{x}\right) = 1 \rightarrow$

$\frac{\pi}{x} = \frac{\pi}{2}, \frac{\pi}{2} + 2\pi, \frac{\pi}{2} + 4\pi, \ldots, \frac{\pi}{2} + 2\pi n, \ldots$

$\Rightarrow \frac{1}{x} = \frac{1}{2}, \frac{1}{2} + 2, \frac{1}{2} + 4, \ldots, \frac{1}{2} + 2n, \ldots$

$= \frac{1}{2}, \frac{5}{2}, \frac{9}{2}, \ldots, \frac{4n+1}{2} \rightarrow x = 2, \frac{2}{5}, \frac{2}{9}, \ldots, \frac{2}{4n+1}$



$\sin\left(\frac{\pi}{x}\right) = -1$

$\Rightarrow \frac{\pi}{x} = \frac{3\pi}{2} + 2n\pi$

$\Rightarrow \frac{1}{x} = \frac{4n+3}{2}$

$\Rightarrow x = \frac{2}{4n+3}, \ n \in \mathbb{Z}$

So, no matter how small we take $\delta$, there's a $\frac{2}{4n+1} < \delta$ & a $\frac{2}{4n+3} < \delta$.

Just take $n$ big enough.

Now back to the limit at $c=0$:

Choose $\varepsilon = \frac{1}{4}$

Suppose $\sin\frac{\pi}{x} \xrightarrow{x \to 0} L$. Then $-1 \le L \le 1$, since $-1 \le f(x) \le 1$

Now, no matter how small $\delta$ is, I can find

$x_1 \in [-\delta, \delta] \ni f(x_1) = 1$ &

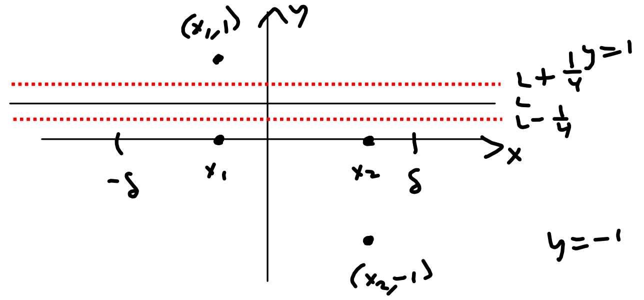
$x_2 \in [-\delta, \delta] \ni f(x_2) = -1$

Then either

$|f(x_1) - L| > \frac{1}{4}$ OR $|f(x_2) - L| > \frac{1}{4} = \varepsilon$

$\varepsilon$-tube:

There's no $\frac{1}{4}$-unit $\varepsilon$-tube that contains both $(x_1, 1)$ & $(x_2, -1)$



Marginal Cost $\quad \dfrac{C(x+1)-C(x)}{1} \approx C'(x) = \lim\limits_{h\to 0} \dfrac{C(x+h)-C(x)}{h}$

cost of producing one additional unit

This week:

§2.1 $\lim\limits_{h\to 0} \dfrac{f(1+h)-f(1)}{h} = f'(1) = \lim\limits_{x\to 1} \dfrac{f(x)-f(1)}{x-1}$

§2.2 $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{x} = f'(x)$

Tangent line

$$y = m(x - x_1) + y_1$$

$$y = m_{tan}(x - x_1) + f(x_1)$$

$$y = \lim_{h \to 0} \frac{f(x_1+h) - f(x_1)}{h} \left( x - x_1 \right) + f(x_1)$$

$$= f'(x_1)(x - x_1) + f(x_1)$$

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Videos have the power rule as a check

$$\frac{d}{dx}\left[f(x)\right] = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$\frac{d}{dx}\left[x^n\right] = n x^{n-1}$$

$$\frac{d}{dx}\left[a f(x)\right] = a \frac{d}{dx}\left[f(x)\right]$$

$$\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right]$$

$$f(x) = 7x^2 - 15x + 1$$

$$\longrightarrow f'(x) = 14x - 15$$

$$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}} \longrightarrow$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{2/3}} = \frac{1}{3\sqrt[3]{x^2}} = \frac{1}{3(\sqrt[3]{x})^2}$$

$$x^{\frac{2}{3}} = x^{\frac{1}{3}\cdot 2} = x^{2\cdot\frac{1}{3}}$$

$$= \left(x^{\frac{1}{3}}\right)^2 = \left(x^2\right)^{\frac{1}{3}}$$

$$= \left(\sqrt[3]{x}\right)^2 = \sqrt[3]{x^2}$$

**Coming Soon to a Calculus Class Near You!**

**The Chain Rule:**

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{d}{dg(x)}\left[f(g(x))\right] \cdot \frac{d}{dx}\left[g(x)\right]$$

$$\frac{d}{dx}\left[\left(x^5\right)^7\right]$$

$$= \frac{d}{dx}\left[f(g(x))\right] = \frac{d}{dx}\left[\left(g(x)\right)^7\right] = 7g(x)^6 \cdot \frac{d}{dx}\left[g(x)\right]$$

$$= 7\left(x^5\right)^6 \cdot 5x^4$$

$$= 35x^{30} \cdot x^4$$

$$= 35x^{34}$$