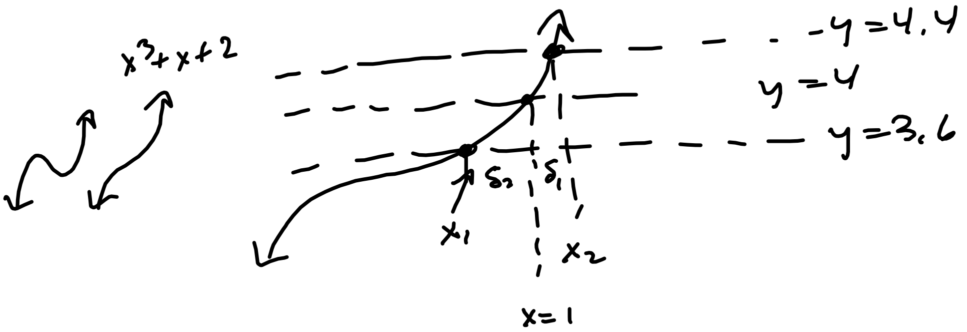
$f(x) = x^3 + x + 2$, $\lim_{x \to 1} f(x) = 4$

$\lim_{x \to c}$ (Polynomial $f(x)$) $= f(c)$, b/c $f(x)$ is cnt's
continuous

(All polynomials are cnt's & smooth)

Let $\varepsilon = 0.4$
Find δ by brute force

ⓐ Solve $f(x) = 4 + .4 = 4.4$
" " $= 3.6 = 4 - .4$



I would just wolframalpha.com part b.
You're doing part a with ε instead of 0.4
general special

Claim:
$$\lim_{x \to 1} (x^3 + x + 2) = 4$$

Scratch work.
$$|f(x) - L| = |x^3 + x + 2 - 4| = |x^3 + x - 2|$$

Split off factor of $x-1$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \\ & x^2 & x^1 & c & r \end{array}$$

sweet!

This says $x^3+x-2=(x-1)(x^2+x+2)$

$= \underbrace{|x-1|}_{\delta}\,|x^2+x+2|$

Need a bound on x^2+x+2 in the neighborhood of $x=1$

Assume $\delta \leq 1$

$\delta \leq 1$ & $x \to 1$ means

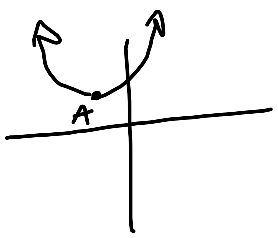
$0 < x < 2$

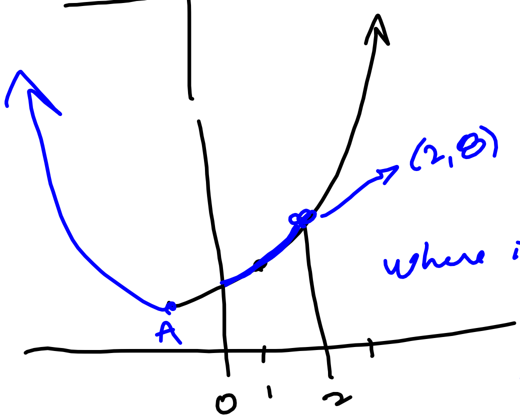
Now look for max $|x^2+x+2|$ on $[0,2]$

$x^2+x+2 = x^2+x+(\frac{1}{2})^2 - \frac{1}{4} + 2$

$= (x+\frac{1}{2})^2 + \frac{7}{4}$

$(h,k) = (-\frac{1}{2}, \frac{7}{4}) = A$





where is $|x^2+x+2|$ biggest here?

$= x^2+x+2$ on $[0,2]$

$2^2+2+2 = 8$ ie

$|x^2+x+2| \le 8$ on $[0,2]$

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{8}\}$

Then $0 < |x-1| < \delta \Longrightarrow$

$|f(x) - L| = |x^3+x+2-4| = |x^3+x-2|$

$= |x-1||x^2+x+2| < \delta \cdot 8 \le \delta \cdot \frac{\varepsilon}{8} = \varepsilon$ $\blacksquare$

§1.8

$f(x)$ is cnt² @ $x = c$ means

$$\boxed{\lim_{x \to c} f(x) = f(c)}$$

There's a lot built into that statement:

f(c) exists.

limit as x approaches c exists

limit as x approaches c agrees with the functional value.