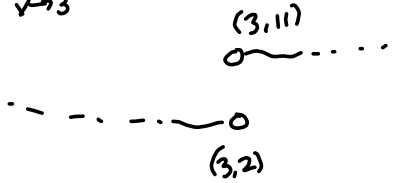
$\lim_{x \to 3^-} f(x) = 2$, $\lim_{x \to 3^+} f(x) = 11$



What's this say about $f(3)$?

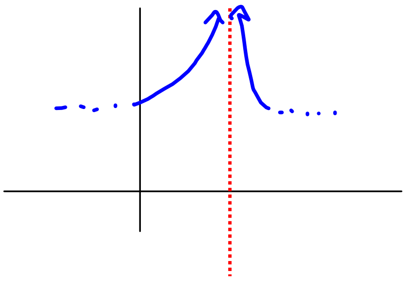
NOTHING !

$\lim_{x \to 3} f(x)$ DNE $\nexists$

Because $\lim_{x \to 3^-} f(x) = 2 \neq 11 = \lim_{x \to 3^+} f(x)$

Left & Right limits must agree.

$\lim_{x \to 3} f(x) = \infty$



∞ ain't a real #.

0 is additive identity

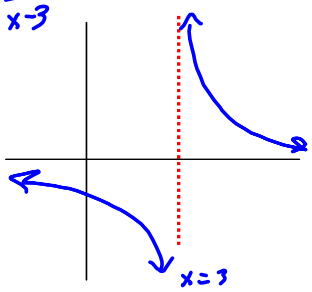
Any # $\neq 0$ added to another # gives a different real #.

$x+1 \neq x$

$1 \neq 0$

$\infty + 1 = \infty$!

$f(x) = \frac{1}{x-3}$



$\lim_{x \to 3^-} (f(x)) = -\infty$

$\lim_{x \to 3^+} (f(x)) = +\infty$

Properties of limits Assume $\lim(f)$, $\lim(g)$ $\exists$ (i.e., are real #s)

$\lim(f+g) = \lim f + \lim g$

$\lim(fg) = (\lim f)(\lim g)$

$\lim(f^n) = (\lim f)^n$ $(n \geq 0)$

If $n < 0$, we need $\lim f \neq 0$ (Division by zero avoided)

$\lim\left(\frac{f}{g}\right) = \frac{\lim(f)}{\lim(g)}$, provided $\lim(g) \neq 0$

$$\lim_{x\to 2} \frac{x^2-3x+2}{x-2} = \lim_{x\to 2} \frac{(x-2)(x-1)}{x-2} = \lim_{x\to 2}(x-1) = 2-1 = 1$$

my way:

$$\frac{x^2-3x+2}{x-2} = \frac{(x-2)(x-1)}{x-2} = x-1 \xrightarrow{x\to 2} 2-1 = 1$$

Need to say $x \neq 1$

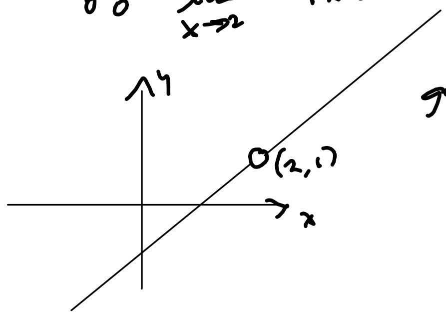
$(x \neq 1)$

$$|f| = \begin{cases} f & \text{if } f \geq 0 \\ -f & \text{if } f < 0 \end{cases}$$

$$\lim_{x\to 2^-}\left(\frac{x^2-3x+2}{|x-2|}\right) = \lim_{x\to 2^-} \frac{(x-2)(x-1)}{-(x-2)} = \lim_{x\to 2^-}\left(\frac{x-1}{-1}\right) = \frac{2-1}{-1} = \frac{1}{-1} = -1$$

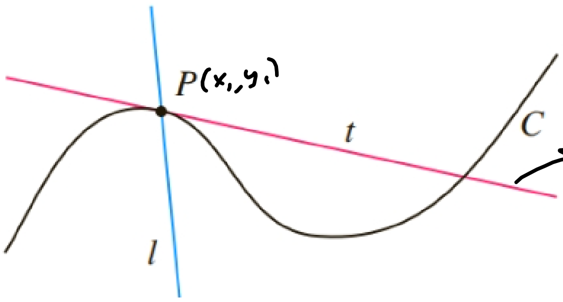
$$\lim_{x\to 2^+}\left(\frac{x^2-3x+2}{|x-2|}\right) = \lim_{x\to 2^+} \frac{(x-2)(x-1)}{x-2} = \lim_{x\to 2^+}\left(\frac{x-1}{1}\right) = 2-1 = 1$$

$\therefore \lim_{x\to 2} \frac{x^2-3x+2}{|x-2|} \nexists$ b/c $1 \neq -1$!

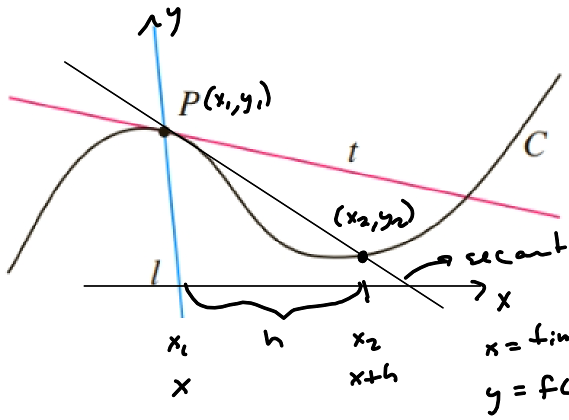


graph of $\frac{x^2-3x+2}{x-2}$

Tangent Problem.



→ tangent line thru (x_1, y_1) with slope $= m_{tan} = \lim_{x_2 \to x_1} \frac{y_2 - y_1}{x_2 - x_1}$



→ secant line with slope $m_{sec} = \frac{y_2 - y_1}{x_2 - x_1}$

x = time (hrs)

$y = f(x)$ = distance (miles)

Lines: Point-Slope: $y - y_1 = m(x - x_1)$ BOOK

$\underline{y = m(x - x_1) + y_1}$ BETTER THAN BOOK.

↳ Accepted (PREFERRED!) on written work.

WebAssign sometimes wants $y = mx + b$, but is actually OK with $y = m(x - x_1) + y_1$ much of the time & $y = m(x - x_1) + y_1$ is less work!

$$m_{tan} = \lim_{x_2 \to x_1} \frac{y_2 - y_1}{x_2 - x_1} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{c \to x} \frac{f(c) - f(x)}{c - x}$$

$$= f'(x)$$

Your book may say $f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

Find $f'(x)$ for $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \left[\left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \right) \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right) \right]$$

$$= \lim_{h \to 0} \frac{x+h-x}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h} + \sqrt{x}\right)} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{1\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}} = f'(x) = m_{tan}}$$

E Find an eq'n of the tangent line to
$f(x)=\sqrt{x}$ @ $x=16$ $\Rightarrow y=f(16)=\sqrt{16}=4 \rightsquigarrow (16,4)$

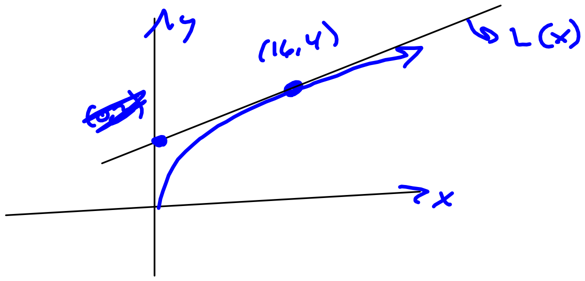
$\boxed{\frac{1}{2\sqrt{x}}=f'(x)=m_{tan}}$ $\Rightarrow y=m_{tan}(x-x_1)+y_1$

$f'(16)=\frac{1}{2\sqrt{16}}=\frac{1}{2(4)}=\boxed{\frac{1}{8}=m_{tan}}$

$\boxed{y=\frac{1}{8}(x-16)+4}$

Sometimes call this the LINEARIZATION of $f(x)$ @ x_1

$L(x)=\frac{1}{8}(x-16)+4$



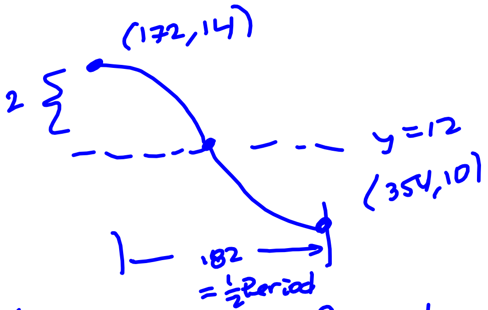
On tests:

Scientific Calculator is all that's allowed.

Of course, I won't be asking those bullshit 1.4 and 1.5 questions.

(Where we plug in a bunch of points and compute slopes (Drill and Kill stuff))

Alex's question about 1.3 #11 (which is #25 in notes and videos!)



$$\begin{array}{r} 354 \\ -172 \\ \hline 182 \end{array}$$

J	31
F	28
M	31
A	30
M	31
J	30
J	31
A	31
S	30
O	31
N	30
D	21

$$\begin{array}{r} 333 \\ 21 \\ \hline 354 \end{array}$$

Period determines coefficient of x, inside.

Period = 365

Want bx inside cosine to give us period of 365 days

cos(bx) wants T = 365

$bx = 2\pi$, when $x = 365$

$365b = 2\pi$

$b = \frac{2\pi}{365}$

$2\cos\left(\frac{2\pi}{365}(x - 172)\right) + 12$

Makes input $= 0$ on June 21^{st}

(172nd day)

To do it w/ sine,

use midline height on the equinox March 21^{st}

J 31

F 128

M 21

80 start

$2\sin\left(\frac{2\pi}{365}(x - 80)\right) + 12$

midline

Amp

Relates to wave-length

Input $= 0$ @ $x = 80$

.. .. .. Makes sine hit its midline.