Course Syllabus Corrected: I got the daggone meeting time wrong.

Course Schedule Pending.

Week 1 Written Assignment Is Posted.

Click on the document of your choice:

Course Syllabus

Course Schedule

Week 1 Written Assignment

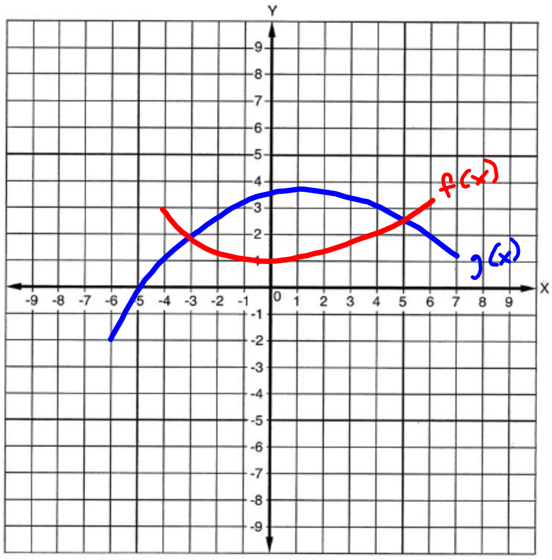
(Click to Open): **Lecture Notes and Videos**

You can surf to all of the above from harryzaims.com:

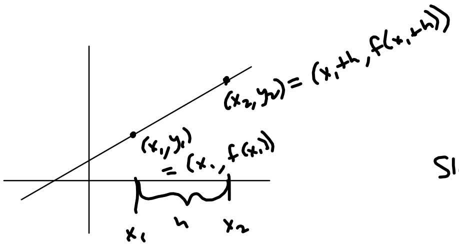
https://harryzaims.com/public_html/

More Specifically, from the Calculus I Page on harryzaims.com (Click to Open):

https://harryzaims.com/public_html/201/2410-spring-25/


Y
X
f(x)
g(x)

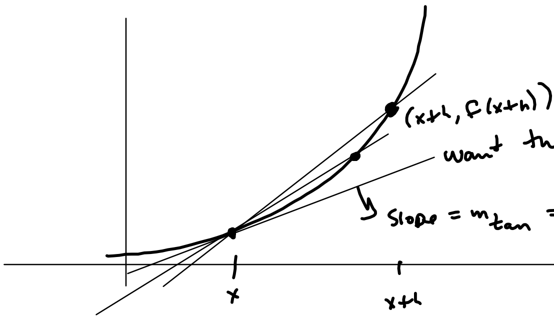
dIFFERENTIAL CALCULUS: WE GENERALIZE THE IDEA OF SLOPE TO GIVE US SLOPE OF A CURVE AT A SINGLE POINT.



$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \text{Differences}$$

$$= \frac{f(x_1+h) - f(x)}{x_1+h - x_1}$$

$$= \frac{f(x+h) - f(x)}{h}$$



want the exact slope @ x.

Slope $= m_{tan}$ = slope of tangent line

As $h \to$ smaller, $\frac{f(x+h) - f(x)}{h} \xrightarrow{h \to 0} m_{tan} = f'(x) =$

Slope of the curve @ x.

$(a+b)^2 = a^2 + 2ab + b^2$

$f(x) = x^2 - 3x + 2 \longrightarrow$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 3(x+h) + 2 - (x^2 - 3x + 2)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 3x - 3h + 2 - x^2 + 3x - 2}{h}$$

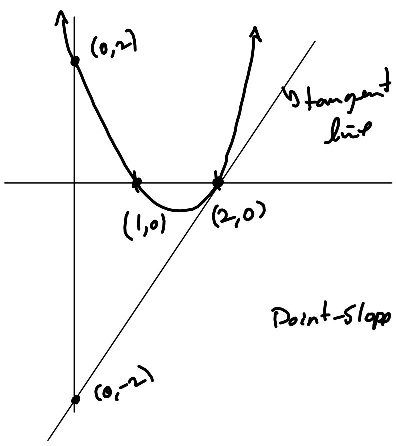
College Algebra ⟵ | ⟶ Calculus.

$$= \frac{2xh + h^2 - 3h}{h} = \frac{h(2x + h - 3)}{h} = 2x + h - 3 \xrightarrow[\text{Limit!}]{h \to 0} \boxed{2x - 3 = f'(x)}$$

$(h \neq 0)$

What's the slope of f(x) @ x = 2?

$x^2 - 3x + 2 = (x-1)(x-2)$



f'(x) gives the slope

so, f'(2) does it!

$f'(x) = 2x - 3 \longrightarrow$

$f'(2) = (2)(2) - 3 = 1$

Tangent Line has slope $m = 1$, passes thru $(2,0) = (x_1, y_1)$

Point-Slope: $y - y_1 = m(x - x_1)$

$y = y_1 + m(x - x_1)$

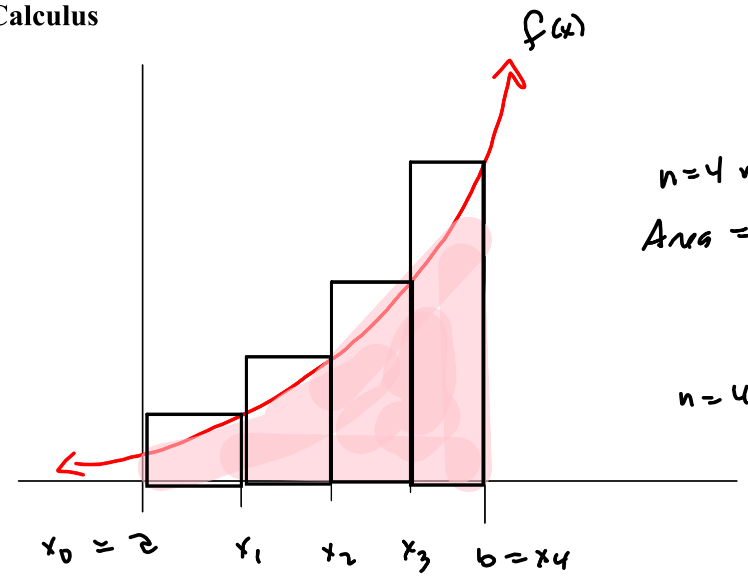
$y = m(x - x_1) + y_1$

$y = 1(x - 2) + 0$

$y = x - 2$ is eq'n of the tangent line.

Integral Calculus

Find Shaded Area.



n = 4 rectangles

$$\text{Area} = \sum_{k=1}^{4} \underbrace{f(x_k)}_{\text{ht.}} \underbrace{(\Delta x)}_{\text{width}}$$

$n = 4 \rightarrow \Delta x = \frac{b-a}{4}$

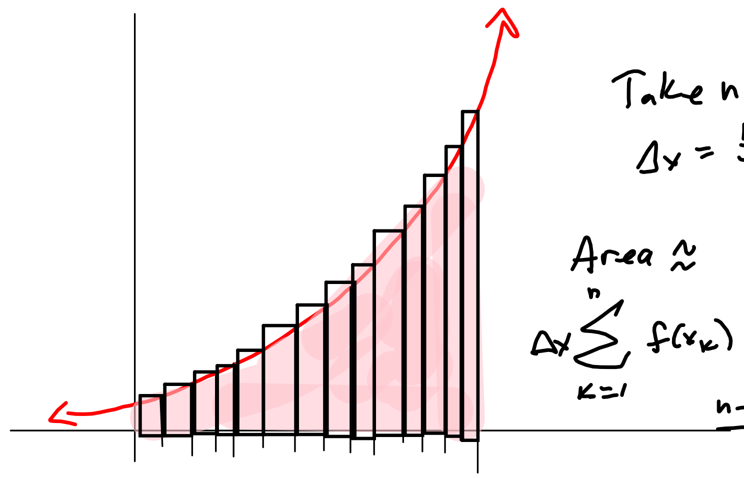
$f(x_k)$:

$x_k = a + k\Delta x = a + \frac{b-a}{4}k$

$$\text{Area} \approx \sum_{k=1}^{4} f\left(a + \frac{b-a}{4}k\right)\left(\frac{b-a}{4}\right)$$

$$= f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x.$$

Find Shaded Area.



Take $n \rightarrow \infty$

$\Delta x = \frac{b-a}{n} \rightarrow 0$

Area $\approx$

$$\Delta x \sum_{k=1}^{n} f(x_k) = \frac{b-a}{n} \sum f\left(a + \frac{b-a}{n}k\right) \xrightarrow{n\to\infty} \text{Area EXACTLY!}$$

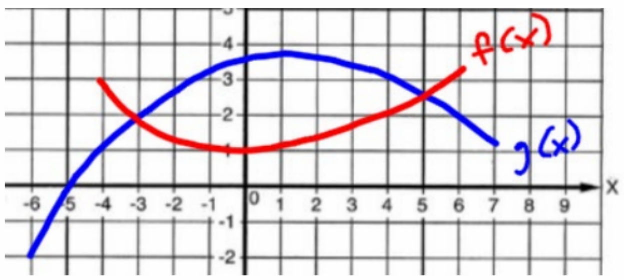
Derivatives need continuous & smooth

$\underbrace{\text{Integrals}}_{\text{weaker condition on } f.}$ need continuous (mostly).

$$\text{Area} = \int_a^b f(x)\,dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=1}^{n} f\left(a + k\left(\frac{b-a}{n}\right)\right)$$

where things didn't come to me as easily as they seemed to come to other people. Usually, I was
k, but when I wasn't, I had to grind. I had to write a lot extra.

swer the following questions about f and g, from the graph:



a. What's the value of $f(0)$?

b. What's the value of $g(6)$?

c. Which is larger, $f(4)$ or $g(4)$?

d. For what values of x is $f(x) = g(x)$?

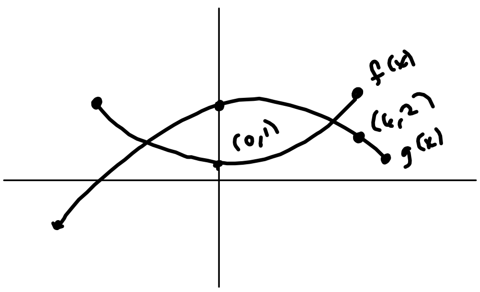
e. On what interval(s) is $f(x) < g(x)$?

f. On what interval(s) is g increasing?

g. On what interval(s) is g decreasing?

h. State the domain and range of f.

i. State the domain and range of g.



a $f(0) = 1$

b $g(6) = 2$

c

Compute diff. quotient for

$f(x) = x^3$:

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^3 - x^3}{h}$$

$$= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

$$= \frac{3x^2h + 3xh^2 + h^3}{h}$$

This is what WebAssign wants in §1.1

$$= \frac{h(3x^2+3xh+h^2)}{h} = \boxed{\underset{(h \neq 0)}{3x^2+3xh+h^2}} \xrightarrow{h \to 0} \boxed{3x^2 = \text{Slope of } x^3 \text{ @ } x.}$$

simplified $\frac{f(x+h)-f(x)}{h}$

This is where we're headed.

$f(x) = -x^3$

$$\frac{f(x+h)-f(x)}{h} = \frac{-(x+h)^3 - (-x^3)}{h}$$

$$= -\frac{(x^3+3x^2h+3xh^2+h^3)+x^3}{h}$$

$$= \frac{-x^3-3x^2h-3xh^2-h^3+x^3}{h}$$

$$= \frac{-3x^2h-3xh^2-h^3}{h} = \frac{h(-3x^2-3xh-h^2)}{h} = \boxed{-3x^2-3xh-h^2}$$

$(h \neq 0)$

Difference Quotient for $\sqrt{x}$. Uses $a^2-b^2=(a-b)(a+b)$

$$\frac{f(x+h)-f(x)}{h} = \left(\frac{\overset{(a}{\sqrt{x+h}} \overset{-\ b)}{-\sqrt{x}}}{h}\right)\left(\frac{\overset{(a+b)}{\sqrt{x+h}+\sqrt{x}}}{\sqrt{x+h}+\sqrt{x}}\right)$$

$$= \frac{\sqrt{x+h}^2-\sqrt{x}^2}{h(\sqrt{x+h}+\sqrt{x})} = \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \boxed{\frac{1}{\sqrt{x+h}+\sqrt{x}}}$$ 1.1 Stuff

$$\boxed{\xrightarrow{h\to 0} \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}} = f'(x)\ !}$$

Later on

$f(x) = \sqrt[3]{x}$

Trick:

$x^3 - y^3 = (x-y)(x^2+xy+y^2)$

$x^3 + y^3 = (x+y)(x^2-xy+y^2)$

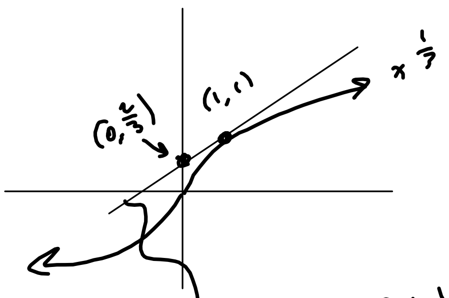
$$\underbrace{\left(\frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h}\right)}_{(a\ -\ b)} \underbrace{\left(\frac{\sqrt[3]{x+h}^2 + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x}^2}{\sqrt[3]{x+h}^2 + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x}^2}\right)}_{(a^2+ab+b^2)}$$

$$\frac{x+h-x}{h\left(\sqrt[3]{x+h}^2 + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x}^2\right)} =$$

$$\frac{\cancel{h}}{\cancel{h}\left(\sqrt[3]{x+h}^2 + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x}^2\right)} = \boxed{\frac{1}{\left(\sqrt[3]{x+h}^2 + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x}^2\right)}}$$

1.1 Stopping point

$$\xrightarrow{h \to 0} \frac{1}{\sqrt[3]{x}^2 + \sqrt[3]{x}\sqrt[3]{x} + \sqrt[3]{x}^2} = \frac{1}{3\left(\sqrt[3]{x^2}\right)} = \boxed{\frac{1}{3x^{2/3}} = f'(x)}$$



$y = \frac{1}{3}(x-1)+1$

$= \frac{1}{3}x + \frac{2}{3}$

Slope @ $x=1$ is

$f'(1) = \frac{1}{3(1)^{2/3}} = \frac{1}{3} = m$

$y = m(x-x_1)+y_1$

$= m(x-x_1)+f(x_1)$

$$\boxed{y = \frac{1}{3}(x-1)+1}$$

= tangent line