MAT 2410 Midterm Test Name_____
100 Points - 2 Hours Spring, 2024

Do your own work on the blank paper provided. Circle final answers (except for the proofs). You may use a scientific calculator, like a TI-30. No graphing calculator. It's better to put a line through your mistakes, rather than doing a lot of erasing.

1. (10 pts) Evaluate $\lim_{x\to -7}\dfrac{x^2+10x+21}{x^2-5x-84}$ by factoring and simplifying.

2. Evaluate each limit, if it exists. If it does not exist give a good reason.

   a. (5 pts) $\lim_{x\to -3^-}\dfrac{x^2-8x-33}{|x+3|}$

   b. (5 pts) $\lim_{x\to -3^+}\dfrac{x^2-8x-33}{|x+3|}$

   c. (5 pts) $\lim_{x\to -3}\dfrac{x^2-8x-33}{|x+3|}$

3. (10 pts) Prove that $\lim_{x\to 2}(13x-11)=15$ (This is the $\varepsilon-\delta$ proof you're dying to do.)

4. (10 pts) Compute the derivative of $f(x)=x^2+5x+6$ by the definition of derivative. This means taking the limit of a difference quotient.

5. (5 pts each) Compute the derivatives of each of the following. Do not simplify your answer.

   a. $y=x^2+5x+\dfrac{6}{x^2}$

   b. $y=\left(x^2-\sin(x)\right)\left(x^2+2x\right)$

   c. $y=\dfrac{x^2+5x}{7x-1}$

   d. $y=\left(x^2\sin(x)\right)^{-4}\left(x^2+2x\right)^3$

   e. $y=\sin\left(\sqrt{\tan\left(x^2+2x\right)}\right)$

6. (5 pts) Find an equation of the tangent line to $f(x)=\tan(x)$ at $x=\dfrac{\pi}{4}$. Then sketch the graph of this situation, with the function and its tangent line, together on the same set of axes.

7. (5 pts) Use your result from the previous problem to approximate $\tan\left(42^0\right)$

8. (10 pts) Find all values of x such that $f\left(x\right)=2\sin\left(x\right)\cos\left(x\right)+x$ has a horizontal tangent.

9. (10 pts) Find $\frac{dy}{dx}$ for the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$.

BONUS SECTION: Are you smarter than the average bear?

B1 (5 pts) Find an equation of the tangent line to the ellipse in #9, above, at the point $\left(x,y\right)=\left(2,\frac{4\sqrt{5}}{3}\right)$.

B2 (5 pts) Prove that $\lim_{x\to 3}\left(x^2-7x+2\right)=-10$.

B3 (5 pts) Use a differential to estimate the change in $f\left(x\right)=\sqrt{x}$ from $x = 4$ to $x = 4.1$. Compare this to the actual change in f over that interval. Which is bigger, dy or Δy? Why?