1) 10pts

$$\frac{x^2+10x+21}{x^2-5x-84} = \frac{(x+7)(x+3)}{(x+7)(x-12)} = \frac{x+3}{x-12} \xrightarrow{x\to -7} \frac{-7+3}{-7-12} = \frac{-4}{-19} = \boxed{\frac{4}{19}}$$

$x \neq -7$

2) a) 5pts

$$\lim_{x\to -3^-} \frac{x^2-8x-33}{|x+3|} = \lim_{x\to -3^-} \frac{(x-11)(x+3)}{-(x+3)} = \lim_{x\to -3^-} \frac{x-11}{-1} = \frac{-3-11}{-1} = \frac{-14}{-1} = \boxed{14}$$

b) 5pts

$$\lim_{x\to -3^+} \frac{x^2-8x-33}{|x+3|} = \lim_{x\to -3^+} \frac{(x-11)(x+3)}{x+3} = \lim_{x\to -3^+} (x-11) = -3-11 = \boxed{-14}$$

c) 5pts

$$f(x) = \frac{x^2-8x-33}{|x+3|} \Longrightarrow \lim_{x\to -3^-} f(x) = 14 \neq -14 = \lim_{x\to -3^+} f(x) \Longrightarrow$$

$$\boxed{\lim_{x\to -3} f(x) \ \nexists}$$

3) 10pts

$$\lim_{x\to 2} (13x-11) = 15.$$ Define $f(x) = 13x-11$.

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{13}$. Then

$0 < |x-2| < \delta \Longrightarrow |f(x) - 15| = |13x-11-15|$

$= |13x-26| = 13|x-2| < 13\delta = 13 \cdot \frac{\varepsilon}{13} = \varepsilon$ ■

4 10 pts $f(x) = x^2 + 5x + 6 \Rightarrow$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 5(x+h) + 6 - (x^2 + 5x + 6)}{h}$$

$$= \frac{x^2 + 2xh + h^2 + 5x + 5h + 6 - x^2 - 5x - 6}{h}$$

$$= \frac{2xh + h^2 + 5h}{h} = \underset{h \neq 0}{2x + h + 5} \xrightarrow{h \to 0} \boxed{2x + 5 = f'(x)}$$

5a 5pts $y = x^2 + 5x + \frac{6}{x^2} \Longrightarrow$ $y = \frac{6}{x^2} = 6x^{-2} \Longrightarrow$ $y' = -12x^{-3}$

$$\boxed{y' = 2x - 5 - 12x^{-3}.}$$

b 5pts $y = (x^2 \sin(x))(x^2+2x) \Longrightarrow$

$$\boxed{y' = (2x - \cos(x))(x^2+2x) + (x^2 \sin(x))(2x+2)}$$

c 5pts $y = \frac{x^2+5x}{7x-1} \Longrightarrow$

$$\boxed{y' = \frac{(2x+5)(7x-1) - (x^2+5x)(7)}{(7x-1)^2}}$$

d 5pts $y = (x^2 \sin(x))^{-4}(x^2+2x)^3 \Longrightarrow$

$$\boxed{\begin{aligned} y' = & -4(x^2\sin(x))^{-5}(2x\sin(x) + x^2\cos(x))(x^2+2x)^3 \\ & + (x^2\sin(x))^{-4}(3)(x^2+2x)^2(2x+2) = f'g + fg' \end{aligned}}$$

e 5pts $y = \sin\left(\sqrt{\tan(x^2+2x)}\right)$

$= \sin\left((\tan(x^2+2x))^{\frac{1}{2}}\right) \Longrightarrow$

$$\boxed{y' = \cos\left((\tan(x^2+2x))^{\frac{1}{2}}\right)\left(\tfrac{1}{2}\cdot \tan(x^2+2x)^{-\frac{1}{2}}(\sec^2(x^2+2x))\right)(2x+2)}$$

6 5pts $f(x) = \tan(x)$. We find $L_{\frac{\pi}{4}}(x)$: $x_1 = \frac{\pi}{4}$

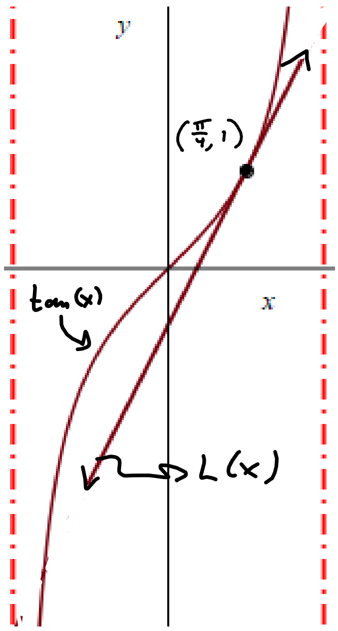
$f(\frac{\pi}{4}) = \tan\left(\frac{\pi}{4}\right) = 1 = y_1 \rightsquigarrow \boxed{(x_1, y_1) = (\frac{\pi}{4}, 1)}$

$f'(x) = \sec^2(x) \Longrightarrow f'(\frac{\pi}{4}) = \sec^2(\frac{\pi}{4}) = (\sqrt{2})^2 = \boxed{2 = m}$

$L_{\frac{\pi}{4}}(x) = f'(\frac{\pi}{4})(x - \frac{\pi}{4}) + f(\frac{\pi}{4})$

$= m(x - x_1) + y_1$

$= \boxed{2(x - \frac{\pi}{4}) + 1 = y}$



7 5pts $x = 42^\circ = 45^\circ - 3^\circ = \frac{\pi}{4} - \frac{\pi}{60}$ radians $\longrightarrow$

$\tan(42^\circ) \approx 2\left(\frac{\pi}{4} - \frac{\pi}{60} - \frac{\pi}{4}\right) + 1 = -\frac{2\pi}{60} + 1 = \boxed{-\frac{\pi}{30} + 1 \approx 0.8952802449}$

Check: $\tan\left(42^\circ \cdot \frac{\pi}{180^\circ}\right) \approx 0.9004040442$

8) 10 pts $f(x) = 2\sin(x)\cos(x) + x$

$\Longrightarrow f'(x) = 2\cos^2(x) - 2\sin^2(x) + 1$

$= 2(1 - \sin^2(x)) - 2\sin^2(x) + 1$

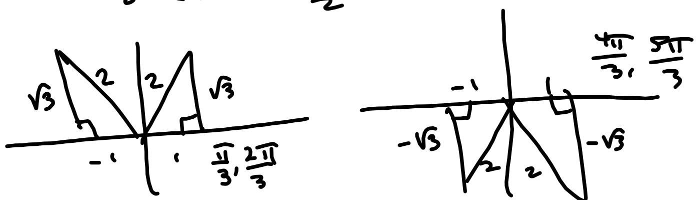
$= 2 - 2\sin^2(x) - 2\sin^2(x) + 1$

$= -4\sin^2(x) + 3 \overset{\text{SET}}{=} 0 \Longrightarrow$

$-4\sin^2(x) = -3 \Longrightarrow$

$\sin^2(x) = \frac{3}{4} \Longrightarrow$

$\sin(x) = \pm\frac{\sqrt{3}}{2}$



Different ways of stating solutions

$S = \{ x + 2n\pi \mid x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3};\ n \in \mathbb{Z} \}$

OR $\{ x + n\pi \mid x = \frac{\pi}{3}, \frac{2\pi}{3} \}$ as solns are π apart.

OR $\frac{\pi}{3} + 2n\pi,\ \frac{2\pi}{3} + 2n\pi,\ \frac{4\pi}{3} + 2n\pi,\ \frac{5\pi}{3} + 2n\pi,\ n \in \mathbb{Z}$.

OR $\frac{\pi}{3} + n\pi,\ \frac{2\pi}{3} + n\pi,\ n \in \mathbb{Z}$

9 10 pts $\frac{x^2}{9}+\frac{y^2}{16}=1 \longrightarrow$

$\frac{2x}{9}+\frac{2yy'}{16}=0 \longrightarrow$ (Divide by 2)

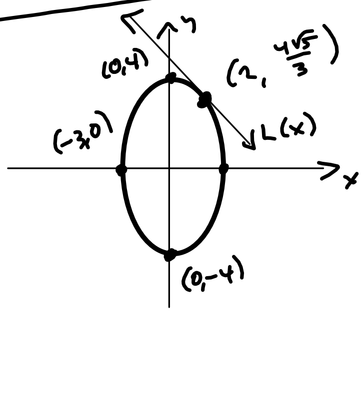
$\frac{yy'}{16} = -\frac{x}{9} \longrightarrow$

$y' = -\frac{x}{9}\cdot\frac{16}{y} = \boxed{-\frac{16x}{9y}=y'}$.

B1 5pts 9 $(x_1,y_1)=\left(2,\frac{4\sqrt{5}}{3}\right) \longrightarrow$

$m = y'\left(2,\frac{4\sqrt{5}}{3}\right) = -\frac{16(2)}{9\left(\frac{4\sqrt{5}}{3}\right)} = -\frac{32}{12\sqrt{5}} = -\frac{16}{6\sqrt{5}} = \boxed{-\frac{8}{3\sqrt{5}}=m}$

$$\boxed{y=L(x)=-\frac{8}{3\sqrt{5}}(x-2)+\frac{4\sqrt{5}}{3}}$$



$-\frac{8}{3\sqrt{5}} = -\frac{8\sqrt{5}}{15}$ also good.

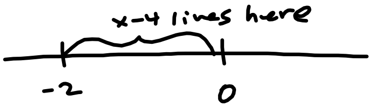
B2 $\lim_{x\to 3}(x^2-7x+2) = -10$

Scratch: $|x^2-7x+2-(-10)|$

$= |x^2-7x+12| = |x-3||x-4|$. Assume $\delta \le 1$.

Then $2 < x < 4 \implies$

$-2 < x-4 < 0 \implies$



$\implies |x-4| < 2 \rightsquigarrow \frac{\varepsilon}{2}$

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{2}\}$. Then $0 < |x-3| < \delta$

$\implies |x^2-7x+2-(-10)| = |x^2-7x+12| = |x-4||x-3|$

$< 2|x-3| < 2\delta \le 2\cdot\frac{\varepsilon}{2} = \varepsilon$ ■

B3 5pts Estimate $\Delta y = \sqrt{4.1} - \sqrt{4}$ with a differential.

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$, $x_1 = 4$, $f(x_1) = \sqrt{4} = 2$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \Rightarrow f'(x_1) = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2} = \frac{1}{4} =$.

$\Rightarrow \Delta y \approx dy = f'(x_1)\,dx = \frac{1}{4} \cdot (.1) = \frac{1}{4} \cdot \frac{1}{10} = \frac{1}{40}$ OR $\boxed{.025 = dy}$

$\Delta y = \sqrt{4.1} - \sqrt{4} \approx \boxed{0.024845673 \approx \Delta y < dy}$

The slope of $f(x)$ is decreasing (concave down) so the tangent line is above the graph.