MAT 2410 Written Final, Spring, 2024 Name____________________
140 Points Final (Chapters 4 – 6)

You know the drill. And remember to circle final answers.

1. (10 pts) Use the limit definition of the definite integral to evaluate $\int_{-1}^{2}\left(x^2+5x\right)dx$. Use a right-endpoint Riemann sum. I don't want you to take it all the way, but I do expect to see the $\Delta x, x_k, f\left(x_k\right)$ written explicitly. Stop just short of actually passing to the limit.

   Bonus (5 pts bonus) Pass to the limit in your answer to #1.

2. Find the area of the region bounded by $y=x^2-4x$ and $y=x$. in two ways.

   a. (5 pts) Sketch the region.

   b. (5 pts) Write the area as an integral with respect to x. Draw a representative rectangle on the sketch from part a.

   c. (5 pts) Evaluate the integral from part b.

   d. (5 pts) Sketch the region again.

   e. (5 pts) Write the area as the sum of two integrals with respect to y. Draw representative rectangles. There will be two different regions, so you will need a rectangle for each region.

   f. (5 pts) Evaluate the sum of integrals from part e.

   g. (5 pts bonus) Compare your results from parts c and f.

   h. (5 pts) Suppose we rotated the region about the line $y=6$. Sketch the graph, and write the integral representing the volume of the solid of revolution obtained. Show a representative disc or washer.

3. We explore absolute value. Let $f\left(x\right)=x^3-4x^2-4x+16$

   a. (5 pts) Provide a rough sketch of $f\left(x\right)$.

   b. (5 pts) Evaluate $\int_0^4 f\left(x\right)dx$.

   c. (5 pts) Provide a rough sketch of $y=\left|f\left(x\right)\right|$.

   d. (5 pts) Evaluate $\int_0^4 \left|f\left(x\right)\right|dx$.

4. Evaluate the indefinite integrals:

   a. (5 pts) $\int (3x+2)^3\, dx$

   b. (5 pts) $\int x^2 (3x+2)^4\, dx$

   c. (5 pts) $\int \sin^4(x)\cos(x)\, dx$

   d. (5 pts) $\int \sin(x)\cdot 2^{\cos(x)}\, dx$

5. Perform the indicated differentiation:

   a. (5 pts) $\dfrac{d}{dx}\int_0^x \dfrac{\cos(2t+1)}{t^2-7}\, dt$

   b. (5 pts) $\dfrac{d}{dx}\int_{\sin(x)}^x \dfrac{\sin(3t)}{t^2+4}\, dt$

6. The function $f(x)=x^2-4x$ is 1-to-1 on the restricted domain $D=[2,\infty)$.

   a. (10 pts) Find the inverse function $f^{-1}(x)$. State its domain and range.

   b. (5 pts) Find $\left(f^{-1}\right)'(5)$, directly, by differentiating your answer for part a.

   c. (5 pts) Find $\left(f^{-1}\right)'(5)$ by applying a theorem regarding derivatives of inverse functions.

7. (5 pts each) Find the derivative with respect to x.

   a. $y=5\cdot 7^{x^2+5x}$

   b. $y=\ln\left(\dfrac{(7x^3-8)^5}{\sqrt{2x\sin(x)}}\right)$

   c. $y=\log_7(x^2-3x)$

   d. $y=\left[\tan(x)\right]^{x^2+4x}$

Bonus Section – Answer any two of the following for up to 10 points.

Bonus 1 (5 pts) Confirm that the hypotheses of the Mean Value Theorem hold for $f(x) = x^3 - 2x^2 + 5x - 1$ on $[0,3]$, and find the c that is promised in the conclusion of the theorem.

Bonus 2 (5 pts) Use the tangent line to approximate $\cos(33^0)$.

Bonus 3 (5 pts) Find $\frac{dy}{dx}$ if $x^2 - 3xy + y^2 = 1$. Then find an equation of the tangent line to the curve at $(1,3)$.

Bonus 4 (5 pts) Evaluate the integral for #2h. You only get credit if your #2h is correct.