

You know the drill. And remember to circle final answers.

1. Ch 4 (10 pts) Use the limit definition of the definite integral to evaluate  $\int_{-1}^2 (2x^2 - x) dx$ . For simplicity, use the limit of the right-endpoint Riemann sum. On the final, I'll be looking for the correct Riemann Sum. Evaluating it will be bonus.
  
2. We find the area of the region bounded by  $y = \sqrt{x}$ ,  $y = x - 2$ , and  $x = 0$  in two ways.
  - a. Ch 3 (5 pts) Sketch the region.
  - b. Ch 4 (5 pts) Write the area as an integral with respect to  $x$ . Draw a representative rectangle on the sketch from part a.
  - c. Ch 4 (5 pts) Evaluate the integral from part b.
  - d. Ch 3 (5 pts) Sketch the region again.
  - e. Ch 4 (5 pts) Write the area as an integral with respect to  $y$ . Draw a representative rectangle on the sketch from part d.
  - f. Ch 4 (5 pts) Evaluate the integral from part e.
  - g. Ch 4 (5 pts bonus) Compare your results from parts c and f.
  - h. Ch 5 (5 pts) Suppose we rotated the region about the line  $x = -1$ . Sketch the graph, and write the integral representing the volume of the solid of revolution obtained. Show a representative disc or washer.
  
3. We explore absolute value. Let  $f(x) = 2 \cos(x) - 1$ 
  - a. Ch 3 (5 pts) Sketch a complete graph of  $f(x)$  on the interval  $[0, 2\pi]$ .
  - b. Ch 4 Evaluate  $\int_0^{2\pi} f(x) dx$ .
  - c. Ch 4 Sketch a complete graph of  $g(x) = |2 \cos(x) - 1| = |f(x)|$  on the interval  $[0, 2\pi]$ .
  - d. Ch 4 Evaluate  $\int_0^{2\pi} g(x) dx$ .
  
4. Evaluate the indefinite integrals:

- a. Ch 4 (5 pts)  $\int (2x-3)^4 dx$
- b. Ch 4 (5 pts)  $\int (2x-3)^4 x^2 dx$
- c. Ch 4 (5 pts)  $\int \sec^4(x)\tan(x)dx$
- d. Ch 6 & Ch 4  $\int \sin(x) \cdot e^{\cos(x)} dx$
5. Ch 4 Suppose I'm pacing back and forth, thinking my usual deep thoughts, and my rate of speed is given by  $r(t)$ , in feet per second. Tell me what the following integrals represent:
- a. (5 pts)  $\int_0^{3600} |r(t)| dt$
- b. (5 pts)  $\int_0^{3600} r(t) dt$
6. Perform the indicated differentiation:
- a. Ch 4 (5 pts)  $\frac{d}{dx} \int_0^x \frac{\sin(3t)}{t^2 + 4} dt$
- b. Ch 4 (5 pts)  $\frac{d}{dx} \int_{x^2}^{\cos(x)} \frac{\sin(3t)}{t^2 + 4} dt$
7. The function  $f(x) = x^2 - 3x + 11$  is 1-to-1 on the restricted domain  $D = \left[\frac{3}{2}, \infty\right)$ .
- a. Ch 6 (10 pts) Find the inverse function  $f^{-1}(x)$ . State its domain and range.
- b. Ch 6 (5 pts) Find  $(f^{-1})'(5)$ , directly, by differentiating your answer for part a.
- c. Ch 6 (5 pts) Find  $(f^{-1})'(5)$  by applying a theorem regarding derivatives of inverse functions.
8. (5 pts each) Find the derivative with respect to  $x$ .
- a. Ch 6  $y = 7 \cdot 5^{x^2-3x}$
- b. Ch 6  $y = \ln \left( \frac{\sqrt[5]{x^2 - 3x}}{(3x^5 + 5x)^3} \right)$

c. Ch 6  $y = \log_5(x^2 - 3x)$

d. Ch 6  $y = [\cos(x)]^{x^2 - 3x}$

9. Ch 6 (10 pts) The half-life of Millsium is 75 years. How old is a Mills skeleton from a burial mound if there is 17% of its natural radioactive Millsium remaining?