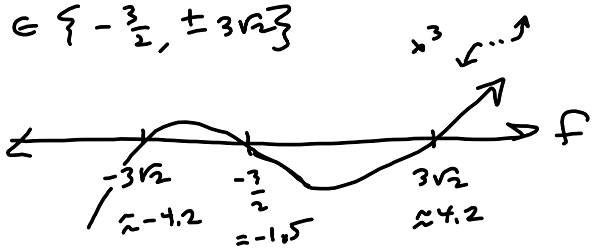
$\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$

$f(x) = 2x^3 + 3x^2 - 36x - 54$

$= x^2(2x+3) - 18(2x+3) = (2x+3)(x^2 - 18) = (2x+3)(x - 3\sqrt{2})(x + 3\sqrt{2}) \overset{\text{SET}}{=} 0$

$\Longrightarrow x \in \{-\frac{3}{2}, \pm 3\sqrt{2}\}$

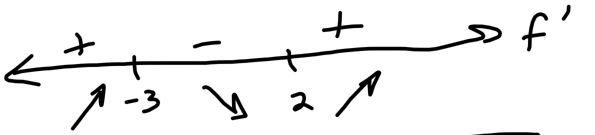


$f(x) = 2x^3 + 3x^2 - 36x - 54 \Longrightarrow$

$f'(x) = 6x^2 + 6x - 36 \overset{\text{SET}}{=} 0 \Longrightarrow$

$x^2 + x - 6 = (x+3)(x-2) = 0 \Longrightarrow$

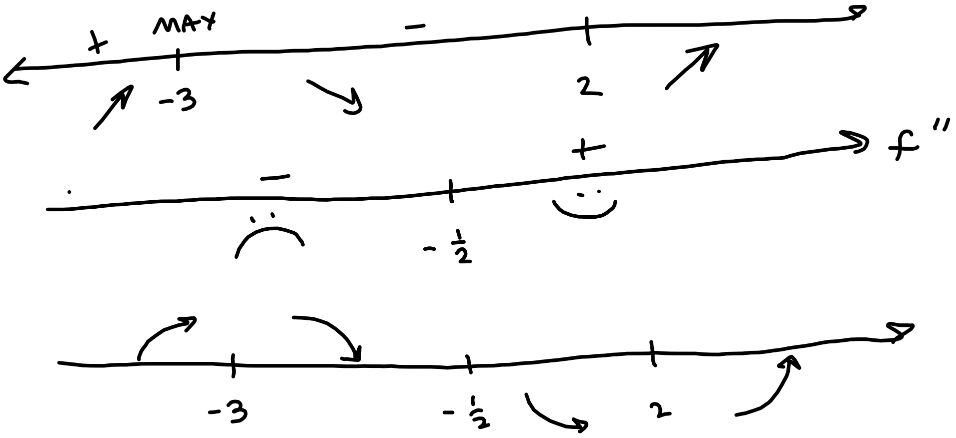
$x \in \{-3, 2\}$



$f''(x) = 12x + 6 \overset{\text{SET}}{=} 0 \Longrightarrow \boxed{x = \frac{-1}{2}}$ I.P.

MIN +

f'



Need:

$f(-3)$, $f(-\frac{1}{2})$, $f(2)$

MAX IP MIN

$f(x) = 2x^3 + 3x^2 - 36x - 54$

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -36 & -54 \\ & & -6 & 9 & 81 \\ \hline & 2 & -3 & -27 & 27 \end{array} = f(-3) \text{ MAX}$$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 3 & -36 & -54 \\ & & -1 & -1 & \frac{37}{2} \\ \hline & 2 & 2 & -37 & -\frac{71}{2} \end{array} = f(-\tfrac{1}{2}) \text{ I.P.}$$

$$-54 + \frac{37}{2} = \frac{-108+37}{2} = -\frac{71}{2} = -35.5$$

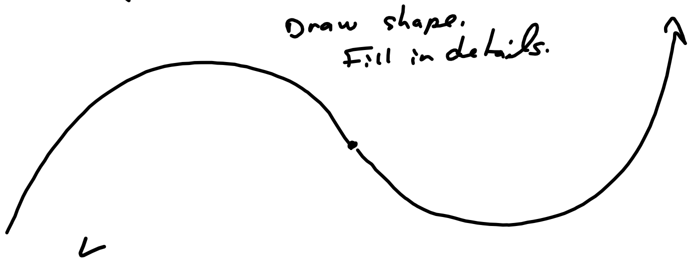
$$\begin{array}{r|rrrr} 2 & 2 & 3 & -36 & -54 \\ & & 4 & 14 & -44 \\ \hline & 2 & 7 & -22 & -98 \end{array} = f(2) \text{ MIN}$$

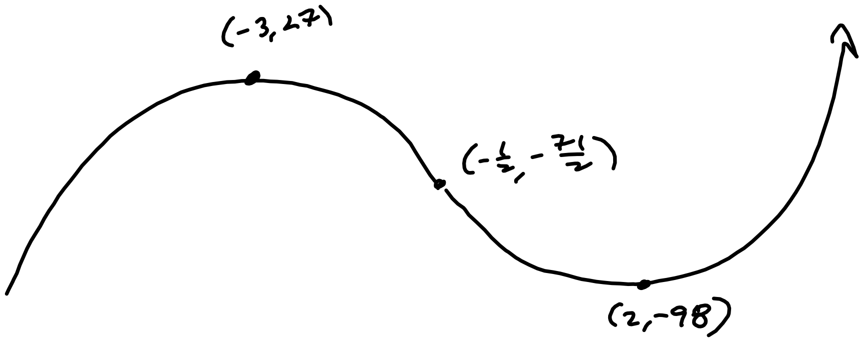
Remind my self of the zeros:

$(\pm 3\sqrt{2}, 0) \approx (\pm 4.2, 0)$

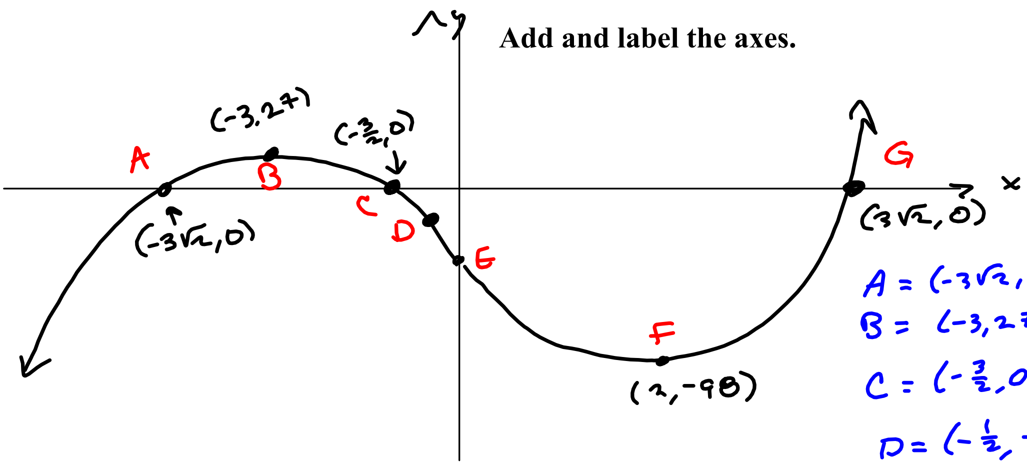
$(-\frac{3}{2}, 0) = (-1.5, 0)$

Draw shape.
Fill in details.





Add and label the axes.



$\pm 3\sqrt{2} \approx \pm 4.242640686$

$A = (-3\sqrt{2}, 0)$ x-int
$B = (-3, 27)$ MAX
$C = (-\frac{3}{2}, 0)$ x-int
$D = (-\frac{1}{2}, -\frac{21}{2})$ IP
$E = (0, -54)$ y-int
$F = (2, -98)$ MIN
$G = (3\sqrt{2}, 0)$ x-int

(2) $R(x) = \dfrac{x^2-4x+3}{x+1}$

$\mathcal{D} = \mathbb{R} \setminus \{-1\}$

$\boxed{\text{V.A.: } x = -1}$

x-int: $R(x) = 0$

$\Longrightarrow x^2-4x+3 = 0$

$\Longrightarrow (x-3)(x-1) = 0$

$\Longrightarrow x = 1, 3 \rightsquigarrow$

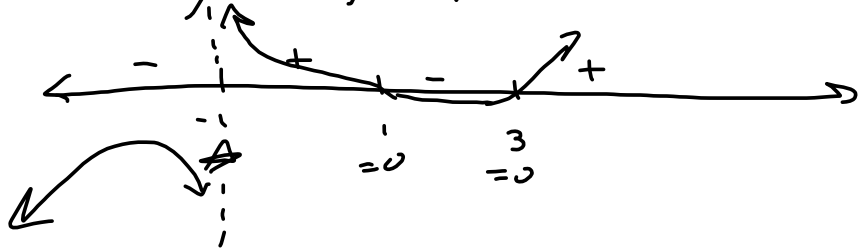
$\boxed{\text{x-int: } (1,0),\ (3,0)}$

Degree of numerator is greater than degree of denominator: Oblique Asymptote. Requires polynomial division. Need the quotient of $x^2-4x+3 \div x+1$:

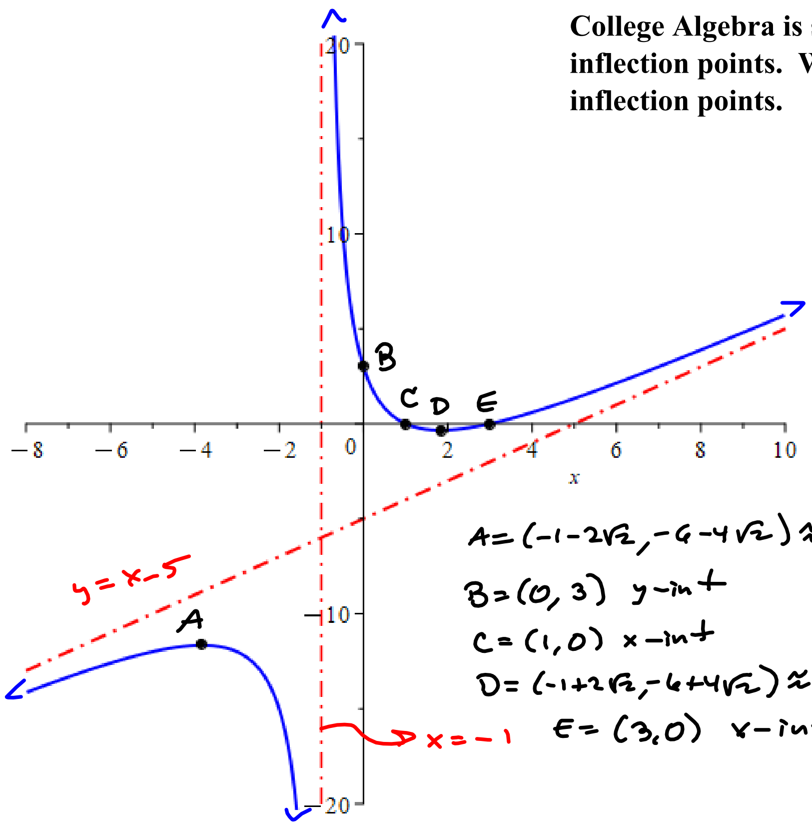
$$\begin{array}{r|rrr} -1 & 1 & -4 & 3 \\ & & -1 & 5 \\ \hline & 1 & -5 & 8 \end{array}$$

$\boxed{y = x-5 \text{ is O.A.}}$

Preliminary (College-Algebra) sketch.



College Algebra is silent on any max/min points or inflection points. We will soon find that there are no inflection points.



$A = (-1-2\sqrt{2}, -6-4\sqrt{2}) \approx (-3.8284, -11.6569)$ MAX

$B = (0, 3)$ y-int

$C = (1, 0)$ x-int

$D = (-1+2\sqrt{2}, -6+4\sqrt{2}) \approx (1.8284, -.3431)$ MIN

$E = (3, 0)$ x-int

Now we do Calculus:

$$R'(x) = \frac{(2x-4)(x+1) - (x^2-4x+3)(1)}{(x+1)^2} = \frac{2x^2-2x-4-x^2+4x-3}{(x+1)^2}$$

$$= \frac{x^2+2x-7}{(x+1)^2} \overset{\text{SET}}{=} 0 \longrightarrow x^2+2x-7=0$$

$$x^2+2x+1^2 = 7+1$$

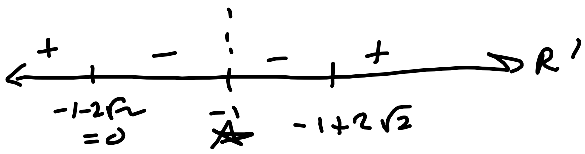
$$(x+1)^2 = 8$$

$$x+1 = \pm\sqrt{8} = \pm 2\sqrt{2}$$

$$x = -1 \pm 2\sqrt{2} \approx$$

$\approx$ 1.828427124, -3.828427124

check sign pattern



$(x+1)^2$ is degree 2. No sign change.

Checks

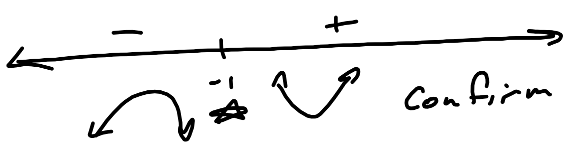
$R'(x) = \dfrac{x^2+2x-7}{(x+1)^2} \Longrightarrow$

$$R''(x) = \frac{(2x+2)(x+1)^2 - (x^2+2x-7)(2(x+1)(1))}{((x+1)^2)^2}$$

$$= \frac{(2x+2)(x+1) - 2(x^2+2x-7)}{(x+1)^3}$$

$$= \frac{2x^2+4x+2-2x^2-4x+14}{(x+1)^3}$$

$$= \frac{16}{(x+1)^3}$$. No IPs. Check concavity:



Confirm College-Algebra guesses.

(3) $g(x) = \sin(2x) + x$

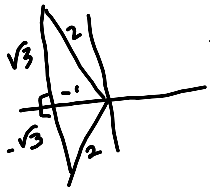
$\mathcal{D} = \mathbb{R}$

$\boxed{y\text{-int}: (0,0)}$ is also the only x-int, by graph.

Max/min:

$g'(x) = 2\cos(2x) + 1 = 0 \longrightarrow$

$\cos(2x) = -\frac{1}{2} \longrightarrow$



$\longrightarrow 2x = \frac{2\pi}{3} + 2n\pi \longrightarrow$

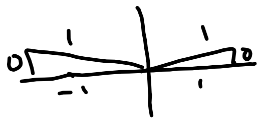
$x = \boxed{\frac{\pi}{3} + n\pi}$ → Local Extrema

$2x = \frac{4\pi}{3} + 2n\pi \longrightarrow x = \boxed{\frac{2\pi}{3} + n\pi}$

$g''(x) = -4\sin(2x) \overset{\text{SET}}{=} 0 \longrightarrow$

$-4\sin(2x) = 0 \longrightarrow$

$\sin(2x) = 0 \longrightarrow$



$2x = 0 + 2n\pi \longrightarrow x = n\pi$

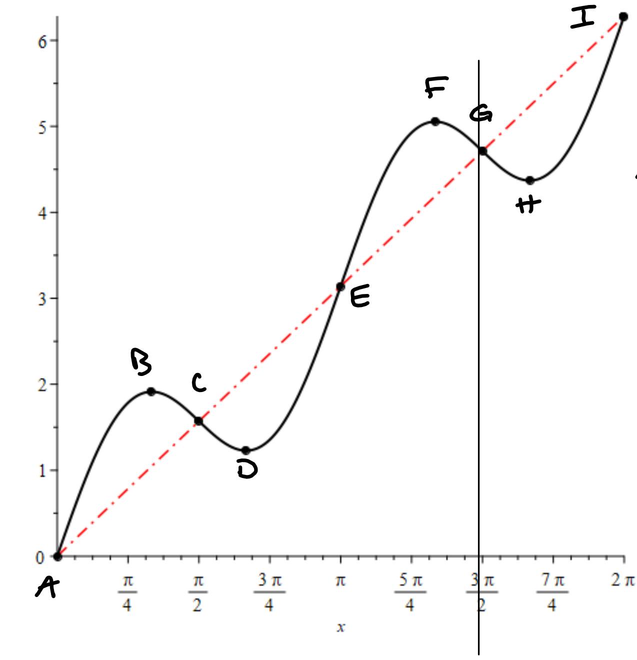
$2x = \pi + 2n\pi \longrightarrow x = \frac{\pi}{2} + n\pi$

Hmmm. $g(n\pi) = \sin(2n\pi) + n\pi = n\pi$

& $g\left(\frac{\pi}{2} + n\pi\right) = \sin\left(2\left(\frac{\pi}{2} + n\pi\right)\right) + \frac{\pi}{2} + n\pi$

$= \sin(\pi + 2n\pi) + \frac{\pi}{2} + n\pi = \frac{\pi}{2} + n\pi$

$\longrightarrow$ All IP's fall on the line $y = x$!



I've only graphed the function over the interval $[0, 2\pi]$.

$A = (0,0)$ IP

$B = \left(\frac{\pi}{3}, \frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$ MAX $\approx (1.047, 1.913)$

$C = \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ IP $\approx (1.571, 1.571)$

$D = \left(\frac{2\pi}{3}, \frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ MIN $\approx (2.094, 1.228)$

$E = (\pi, \pi)$ IP $\approx (3.142, 3.142)$

$F = \left(\frac{4\pi}{3}, \frac{4\pi}{3} + \frac{\sqrt{3}}{2}\right)$ MAX $\approx (4.189, 5.055)$

$G = \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$ IP $\approx (4.712, 4.712)$

$H = \left(\frac{5\pi}{3}, \frac{5\pi}{3} - \frac{\sqrt{3}}{2}\right)$ MIN $\approx (5.236, 4.370)$

$I = (2\pi, 2\pi)$ IP $\approx (6.283, 6.283)$

Scratch Work Building #3.

$g(x) =$

This is a way to engineer max/mins where I want them.

$$g'(x) = 4\sin^2(x) - 3$$

$$\hat{g}(x) = \int_0^x (4\sin^2(x) - 3)\,dx$$

$$= \int_0^x \left(4\left(\frac{1-\cos(2x)}{2}\right) - 3\right)dx$$

$$= \int_0^x (2 - 2\cos(2x) - 3)\,dx$$

$$= \int_0^x (-2\cos(2x) - 1)\,dx$$

$$= -\int_0^x 2\cos(2x)\,dx - \int_0^x dx$$

$$= -\int_0^x \cos(2x)\cdot 2\,dx - x$$

$$\hat{g}(x) = -\sin(2x) - x$$

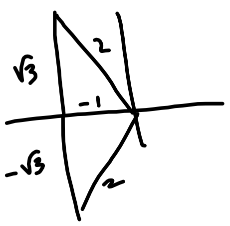
$$\hat{g}'(x) = -2\cos(2x) - 1 \overset{SET}{=} 0$$

$0 \le x \le 2\pi \Longrightarrow 0 \le 2x \le 4\pi$

$\Longrightarrow \cos(2x) = -\frac{1}{2}$

$2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3} \Longrightarrow$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ ✓



OR $-2(2\cos^2(x) - 1) - 1$

$= -4\cos^2(x) + 2 - 1$

$= -4\cos^2(x) + 1 = 0$

$= -4(1 - \sin^2(x)) + 1 = 0$

$= -4 + \sin^2(x) + 1$

$= 4\sin^2(x) - 3 = 0$

$\sin(x) = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$