1. (10 pts) $\lim_{x\to 3}\frac{x^2-9}{x^3-27}$ : $\lim_{x\to 3}\frac{(x-3)(x+3)}{(x-3)(x^2+3x+9)}$

$= \lim_{x\to 3}\frac{x+3}{x^2+3x+9} = \frac{3+3}{3^2+3\cdot 3+9} = \frac{6}{27} = \boxed{\frac{2}{9}}$

2a. (5 pts ) $\lim_{x\to -3}\frac{2x^2+x-15}{x^2+5x+6}$ : $\frac{2x^2+x-15}{x^2+5x+6} = \frac{(2x-5)(x+3)}{(x+2)(x+3)}$

$= \underset{(x\neq -3)}{\frac{2x-5}{x+2}} \xrightarrow{x\to -3} \frac{2(-3)-5}{-3+2} = \frac{-11}{-1} = \boxed{11}$

2b.. (5 pts ) $\lim_{x\to -3}\frac{2x^2-x-15}{x^2+5x+6}$ : $\frac{2x^2-x-15}{x^2+5x+6} = \frac{(2x+5)(x-3)}{(x+3)(x+2)}$

Nothing cancels. $x\to -3$ is impossible.

$2x^2-x-15$ :
$b^2-4ac = 1-4(2)(-15)$
$= 1+120 = 121$ FACTORS $ac = -30 =$ magic

middle term: $-1 = -2+1$ $\quad -2$
$= -6+5$ $\quad -30$ ✓
$2x^2-6x+5x-15$
$= 2x(x-3)+5(x-3)$
$= (x-3)(2x+5)$

3. $\lim_{x \to 5}(2x-7)=3$

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{2}$.

Then $0 < |x-5| < \delta \implies |2x-7-3| = |2x-10| = 2|x-5| < 2\delta = \varepsilon$ ■

4. (10 pts) $f(x) = x^2 + 5x + 6 \implies$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+5(x+h)+6-(x^2+5x+6)}{h}$$

$$= \frac{x^2+2xh+h^2+5x+5h+6-x^2-5x-6}{h}$$

$$= \frac{2xh+h^2+5h}{h} \underset{h \neq 0}{=} 2x+h+5 \xrightarrow{h \to 0} \boxed{2x+5=f'(x)}$$

5. a. (5 pts) $y = x^2 + 5x + \frac{6}{x^2}$ $= x^2 + 5x + 6x^{-2} \Longrightarrow$

$\boxed{y' = 2x + 5 - 12x^{-3}}$

b. (5 pts) $y = (x^2 + 5x)(7x - 1)$ $\Longrightarrow$

$\boxed{y' = (2x+5)(7x-1) + (x^2+5x)(7)}$

c. (5 pts) $y = \frac{x^2 + 5x}{7x - 1}$ $\Longrightarrow$

$$\boxed{y' = \frac{(2x+5)(7x-1) - (x^2+5x)(7)}{(7x-1)^2}}$$

d. (5 pts) $y = (x^2 + 5x)^3 (7x - 1)^5$ $\Longrightarrow$

$\boxed{y' = 3(x^2+5x)^2(2x+5)(7x-1)^5 + (x^2+5x)^3(5(7x-1)^4)(7)}$

$= (x^2+5x)^2(7x-1)^4\left[3(2x+5)(7x-1) + 35(x^2+5x)\right]$ NATE MARTINEZ

$= (x^2+5x)^2(7x-1)^4\left[3(14x^2+33x-5) + 35x^2 + 175x\right]$

$= (x^2+5x)^2(7x-1)^4\left[42x^2 + 99x - 15 + 35x^2 + 175x\right]$

$= (x^2+5x)^2(7x-1)^4\left[77x^2 + 274x - 15\right]$

e. (5 pts) $y = \cot(\sec(x^2 - 5))$ $\Longrightarrow$

$\boxed{y' = -\csc^2(\sec(x^2-5))(\sec(x^2-5)\tan(x^2-5))(2x)}$

6. (10 pts) Find an equation of the tangent line to $f(x) = \sin(x)$ at $x = \frac{\pi}{3}$. Then sketch the graph of this situation, with the function and its tangent line, together on the same set of axes.

$f(x) = \sin(x)$, $x_1 = \frac{\pi}{3}$. $\longrightarrow$

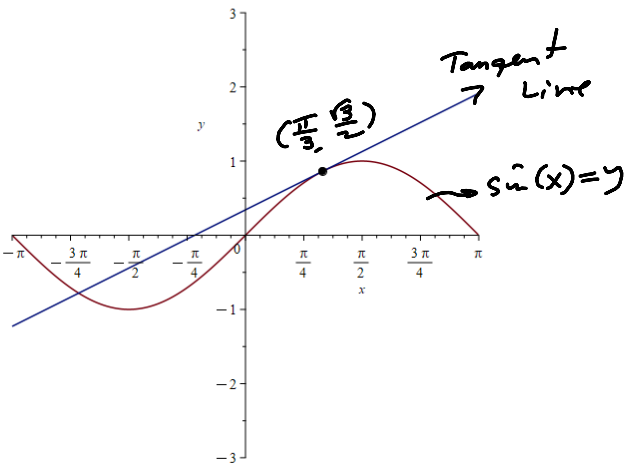
$f(x_1) = y_1 = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$, $\&$

$f'(x_1) = \cos(x_1) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

Tangent Line is

$y = f'(x_1)(x - x_1) + y_1$

$$\boxed{y = \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}}$$



7. (5 pts) Use your result from the previous problem to approximate $\sin(65^\circ)$

$\frac{\pi}{3} = 60^\circ$

$65^\circ = 60^\circ + 5^\circ = \frac{\pi}{3} + (5^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{3} + \frac{\pi}{36}$

$\Delta x = \frac{\pi}{36} = x - x_1$

$y = L_{\frac{\pi}{3}}(x) = \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2} \Longrightarrow$

$L_{\frac{\pi}{3}}(x) = \frac{1}{2}\left(\frac{\pi}{3} + \frac{\pi}{36} - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$

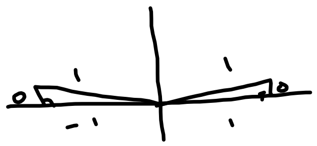
$= \frac{1}{2}\left(\frac{\pi}{36}\right) + \frac{\sqrt{3}}{2}$

$= \boxed{\frac{\pi}{72} + \frac{\sqrt{3}}{2} \approx \sin(65^\circ)}$ 0.9096586353

**Actual:** $\sin(65^\circ) \approx$ 0.9063077871

8. (10 pts) Find all values of $x$ such that $f(x) = 1 + 2\cos(x)$ has a horizontal tangent.

$f'(x) = -2\sin(x) \overset{\text{SET}}{=} 0$



$$\underbrace{x = 0 + 2\pi n \quad \text{OR} \quad x = \pi + 2\pi n}_{\text{combine!}}$$

$x = \pi n.$

$$\boxed{\therefore \{x \mid f'(x) = 0\} = \{\pi n \mid n \in \mathbb{Z}\}}$$

9. (10 pts) Find $\frac{dy}{dx}$, given that $\sec x + \sin y = 2xy - 3x^2y^2$

$\sec(x)\tan(x) + \cos(y)y' = 2y + 2xy' - 6xy^2 - 6x^2yy' \longrightarrow$

$(\cos(y) - 2x + 6x^2y)y' = 2y - 6xy^2 - \sec(x)\tan(x) \longrightarrow$

$$\boxed{y' = \frac{2y - 6xy^2 - \sec(x)\tan(x)}{\cos(y) - 2x + 6x^2y}}$$

B1 (5 pts) Find the derivative of $f(x)=\frac{1}{\sqrt{x}}$ , by the definition of the derivative.

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left[f(x+h)-f(x)\right]$$

$$= \frac{1}{h}\left[\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right] = \frac{1}{h}\left[\frac{1}{\sqrt{x+h}}\cdot\frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}}\cdot\frac{\sqrt{x+h}}{\sqrt{x+h}}\right]$$

$$= \frac{1}{h}\left[\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right] = \frac{1}{h}\left[\frac{(\sqrt{x}-\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}\right]$$

$$= \frac{1}{h}\left[\frac{x-(x+h)}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}\right] = \frac{1}{h}\left[\frac{-h}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}\right]$$

$$= \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})} \xrightarrow{h\to 0} \frac{-1}{x(2\sqrt{x})} = \boxed{\frac{-1}{2x\sqrt{x}} = f'(x)} = -\frac{1}{2x^{3/2}}$$

$(h \neq 0)$

B2 (5 pts) Prove that $\lim_{x\to 3}\left(x^2+5x+2\right)=26$

Scratch: $|x^2+5x+2-26| = |x^2+5x-24| = |x+8||x-3|$

$\delta < 1 \Rightarrow 2 < x < 4 \Rightarrow$

$10 < x+8 < 12$

$\Rightarrow |x+8| < 12$ ✓

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \min\left\{1, \frac{\varepsilon}{12}\right\}$.

Then $0 < |x-3| < \delta \Rightarrow |x^2+5x+2-26| = |x^2+5x-24|$

$= |x+8||x-3| < 12|x-3| < 12\delta \leq 12\cdot\frac{\varepsilon}{12} = \varepsilon$ ∎