Bonus Question $\lim_{x \to -3}\left(x^2 - 2x + 5\right) = 20$

$$\lim_{u \to -5} \frac{u+5}{u^3+125} = \lim_{x \to -5} \frac{u+5}{(u+3)(u^2-3u+25)}$$

$$= \lim_{u \to -5} \frac{1}{u^2-3u+25} = \frac{1}{25+15+25} = \boxed{\frac{1}{65}}$$

$$\left(a^3+b^3\right) = (a+b)(a^2-ab+b^2)$$

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

$y = 3 \cdot 5^{\sin(x)}$

$\Rightarrow y' = 3 \cdot \ln(5) \cdot 5^{\sin(x)} \cdot \cos(x)$

$y = \ln\left(\frac{\sin^2(x)}{x^2(\cos(x)+1)}\right) = 2\ln(\sin(x)) - 2\ln(x) - \ln(\cos(x)+1)$

$\Rightarrow y' = \frac{2\cos(x)}{\sin(x)} - \frac{2}{x} + \frac{\sin(x)}{\cos(x)+1}$

$y = \log_{11}\left(\frac{\sin(x)}{\cos(x)+1}\right) = \log_{11}(\sin(x)) - \log_{11}(\cos(x)+1)$

$= \frac{\ln(\sin(x))}{\ln(11)} - \frac{\ln(\cos(x)+1)}{\ln(11)} \Rightarrow$

$\Rightarrow y' = \frac{1}{\ln(11)}\left(\frac{\cos(x)}{\sin(x)}\right) + \frac{1}{\ln(11)}\left(\frac{\sin(x)}{\cos(x)+1}\right)$

$y = (7x+1)^{\tan(x)}$

$\ln(y) = \tan(x)\ln(7x+1)$

$\frac{y'}{y} = \sec^2(x)\ln(7x+1) + \tan(x)\left(\frac{7}{7x+1}\right)$

$\Rightarrow y' = \left(\sec^2(x)\ln(7x+1) + \frac{7\tan(x)}{7x+1}\right)(7x+1)^{\tan(x)}$

JQ

Scientific Calculator - Yes
Graphing Calculator - No

Claim $f(x) = x^2 - 5x + 2$ Claim $\lim_{x \to 3} f(x) = -4 = L$

$\lim_{x \to 3} (x^2 - 5x + 2) = -4 = L$

scratch

Want $|x^2 - 5x + 2 - (-4)| = |x^2 - 5x + 6| = |x-2||x-3|$ WANT $< \varepsilon$

Assume $\delta \le 1 \implies |x-3| < \delta \le 1$

$\implies |x-3| < 1 \implies$

$$\begin{array}{r} -1 < x-3 < 1 \\ 1 = \quad +1 = 1 \\ \hline 0 < x-2 < 2 \end{array}$$

$\implies |x-2| < 2$

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{2}\}$.

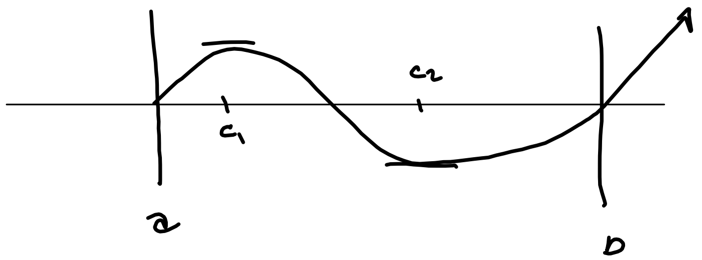
Then $0 < |x-3| < \delta \implies |f(x) - L| = \ldots = |x-2|\,|x-3| < 2\delta \le 2\frac{\varepsilon}{2}$

$= \varepsilon$ ∎

Rolle's

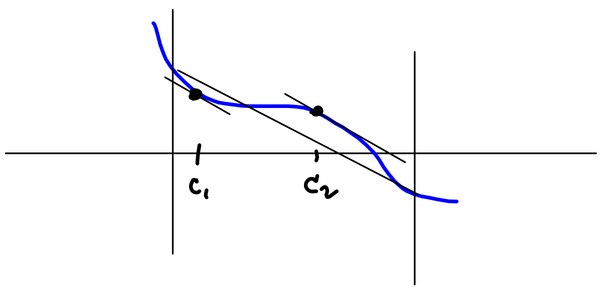
f cnt^s on $[a,b]$, difb^l on (a,b) & $f(a) = f(b) \Longrightarrow$

$\exists\, c \in (a,b) \ni f'(c) = 0$.



MVT

f cnt^s on $[a,b]$, difb^l on $(a,b) \Longrightarrow \exists\, c \in (a,b) \ni f'(c) = \dfrac{f(b) - f(a)}{b-a}$



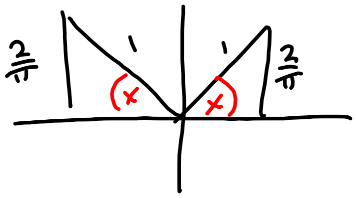
$y = \sin(x)$ on $[0, \frac{\pi}{2}]$

Confirm this function satisfies the hypotheses of MVT and find the *c* that is promised.

$\sin(x)$ is cnt^s & difb^l everywhere, so cnt^s on $[a,b]$ & difb^l on (a,b).

$\sin(0) = 0$, $\sin(\frac{\pi}{2}) = 1 \longrightarrow \frac{f(b)-f(a)}{b-a} = m_{AVG} = \frac{1-0}{\frac{\pi}{2}-0} = \frac{2}{\pi}$

$f'(x) = \cos(x) \overset{SET}{=} \frac{2}{\pi}$



$x \in (0, \frac{\pi}{2})$

$\cos(x) = \frac{2}{\pi}$

$\boxed{x = \arccos\left(\frac{2}{\pi}\right) = c}$

Same Question on $[\frac{\pi}{2}, \pi]$

$\sin(\frac{\pi}{2}) = 1$
$\sin(\pi) = 0$

$m_{avg} = \frac{f(\pi) - f(\frac{\pi}{2})}{\pi - \frac{\pi}{2}} = \frac{0-1}{\frac{\pi}{2}} = -\frac{2}{\pi}$

so $\cos(x) = -\frac{2}{\pi}$

$x = \arccos\left(-\frac{2}{\pi}\right)$

$= \pi - \arccos\left(\frac{2}{\pi}\right)$

$f(x) = \cos(x)$ on $[\pi, \frac{3\pi}{2}]$

$\frac{\cos(\frac{3\pi}{2}) - \cos(\pi)}{\frac{3\pi}{2} - \pi} = \frac{-1}{\frac{\pi}{2}} = -\frac{2}{\pi}$

$\cos(x) = -\frac{2}{\pi}$

$x = \arccos\left(-\frac{2}{\pi}\right)$?

Not quite. $x \in Q\,\text{III}$, not $Q\,\text{II}$

$c = 2\pi - \arccos\left(-\frac{2}{\pi}\right)$