No epsilon-delta on the test. The final's already out there.

I'd be fine adding a bonus epsilon-delta that you can bring with you on your cheat sheet.

Prove that $\lim_{x\to 3}(x^2-5x+2) = -4 = L$

Scratch:

$$|f(x)-L| = |x^2-5x+2-(-4)| = |x^2-5x+6| = |(x-2)\underbrace{(x-3)}_{<\delta}|$$

$< \delta|x-2|$ want $< \varepsilon$

Get a bound on $|x-2|$ in vicinty of $x=3$

$|x-3| < \delta \Rightarrow$

$-\delta < x-3 < \delta$

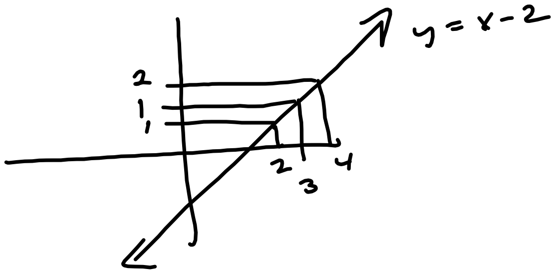
$-\delta+1 < x-2 < \delta+1$

$0 = -1+1 < x < \delta+3 < 1+3$

$\cancel{\delta > -1}$ d

$-\delta > -1 \Rightarrow$

$\delta < 1$



$\delta \le 1 \Rightarrow$
$-\delta \ge -1$

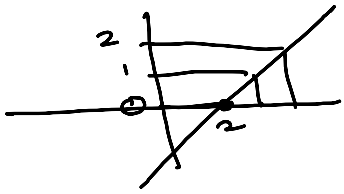
Assume $\delta \le 1$

$0 < x < 4$

So,

$-2 < x-2 < 2$

$\Rightarrow \boxed{|x-2| < 2}$



$-5 < x < 7$

$-7 < x-2 < 5$

$\Rightarrow |x-2| < 7$

$\delta |x-2| < 2\delta \overset{\text{want}}{<} \varepsilon \Rightarrow$

Let $\delta = \min\{1, \frac{\varepsilon}{2}$

$|x^2-5x+2-(-4)| = |x^2-5x+6| = |x-3||x-2|$

$< |x-3| \cdot 2 \quad < 2\delta \le 2 \cdot \frac{\varepsilon}{2} = \varepsilon$

$\delta = \min\{1, \frac{\varepsilon}{2}\}$

$f(x) = x^2 - 5x + 2 \xrightarrow{x \to 3} -4 = L$

Proof

Let $\epsilon > 0$ be given. Define $\delta = \min\{1, \frac{\epsilon}{2}\}$. Then if $0 < |x-3| < \delta$, then $|f(x) - L| = |x^2 - 5x + 2 - (-4)|$

$= |x^2 - 5x + 6| = |x-3||x-2| < 2|x-3| < 2\delta \leq 2\left(\frac{\epsilon}{2}\right) = \epsilon$ ∎

BONUS Question Like this

I'll tell you in class, tomorrow.

$f(x) = x^2 - 5x + 2 \xrightarrow{x \to 3} -4 = L$

Want $|f(x) - L| = |x^2 - 5x + 2 - (-4)| = |x^2 - 5x + 6| < |x-3||x-2| < \varepsilon$

Need bound on $|x-2|$

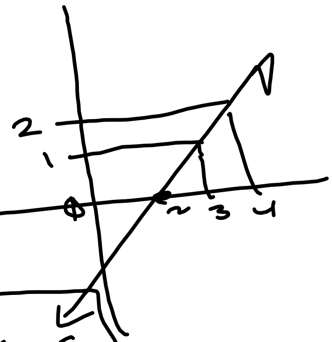
Use $x \to 3$ & assume $\delta \le 1$

$\Rightarrow 0 < |x-3| < \delta \le 1$

$\Rightarrow 2 < x < 4$

$0 = 2-2 < x-2 < 2$

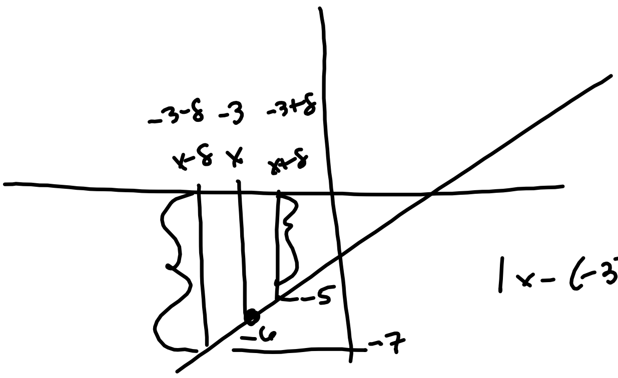
$\Rightarrow |x-2| < 2$



Proof Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{2}\}$

If $0 < |x-3| < \delta$, Then $|f(x) - L| = \dots = |x-3||x-2|$

$< \delta \cdot 2 = 2\delta \le 2 \cdot \frac{\varepsilon}{2} = \varepsilon$ ∎



$|x-(-3)| = |x+3| < 7$ if $\delta \le 1$

See Tuesday's video/notes from after class. Logarithmic Differentiation.

$y = (x^2-5x)^{x^2-5x}$

$\frac{d}{dx}\left[\ln(y) = (x^2-5x)\ln(x^2-5x)\right]$

$\frac{y'}{y} = (2x-5)\ln(x^2-5x) + \cancel{(x^2-5x)}\left(\frac{2x-5}{\cancel{x^2-5x}}\right) \longrightarrow$

$y' = \left((2x-5)(\ln(x^2-5x)) + 2x-5\right)(x^2-5x)^{x^2-5x}$

Approximate $\tan(47^\circ)$ using tangent line

$x_1 = 45^\circ$ b/c we know $\tan(45^\circ)$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\sec\theta = \sqrt{2}$

$f(x) = \tan(x) \Rightarrow f(x_1) = \tan(45^\circ) = 1$

$f'(x) = \sec^2(x) \Rightarrow f'(x_1) = \sec^2(45^\circ) = \sqrt{2}^2 = 2 = f'(x_1)$

$\Rightarrow L(x) = f'(x_1)(x - x_1) + f(x_1)$

$= 2(x - 45^\circ) + 1$

$\Rightarrow L(47^\circ) = 2(47 - 45) + 1$

$= 2(2) + 1 = 5$

$\boxed{\text{Teacher BAD. Use radians}}$

$45^\circ = \frac{\pi}{4}$

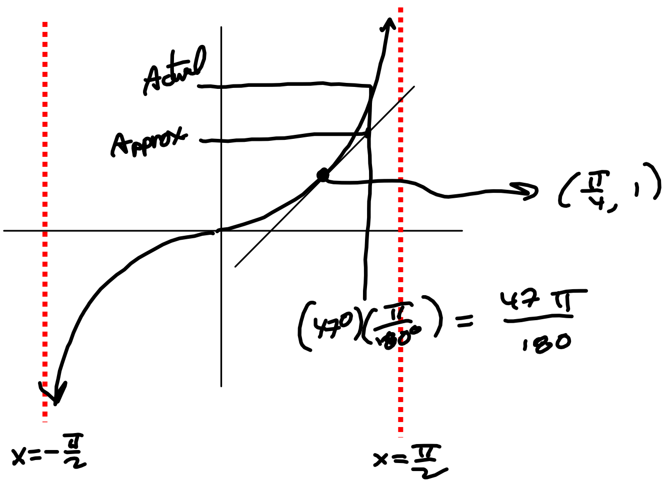
$47^\circ = \frac{\pi}{4} + 2^\circ = \frac{\pi}{4} + 2\left(\frac{\pi}{180}\right) = \frac{\pi}{4} + \frac{\pi}{90} \rightarrow$

$L\left(\frac{\pi}{4} + \frac{\pi}{90}\right) = 2\left(\frac{\pi}{4} + \frac{\pi}{90} - \frac{\pi}{4}\right) + 1 = 2\left(\frac{\pi}{90}\right) + 1$

$= \frac{\pi}{45} + 1$

Actual: $\tan(47^\circ) \approx 1.072368710$ ≈ 1.069813170

Why's $f(47^\circ) >$ Approximation?



$(47^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{47\pi}{180}$

Implicit Differentiation

$\frac{d}{dx}\left[x^2 - 5xy + 2\sin(x)\cos(y) = 1 \right]$

Find $\frac{dy}{dx}$:

$2x - 5y - 5xy' - 5x + 2\cos(x)\cos(y) - 2\sin(x)\sin(y)\,y' = 0$

$\Rightarrow \left(-5x - 2\sin(x)\sin(y)\right) y' = -2x + 5y + 5x - 2\cos(x)\cos(y)$

$$\boxed{y' = \frac{-2x + 5y + 5x - 2\cos(x)\cos(y)}{-5x - 2\sin(x)\sin(y)}}$$

$5xy = 5fg$

$(5fg)' = 5(f'g + fg')$

MVT for $f(x)=x^2-5x+1$ on $[0,1]$

$f(0)=1$ & $f(1)=-3$

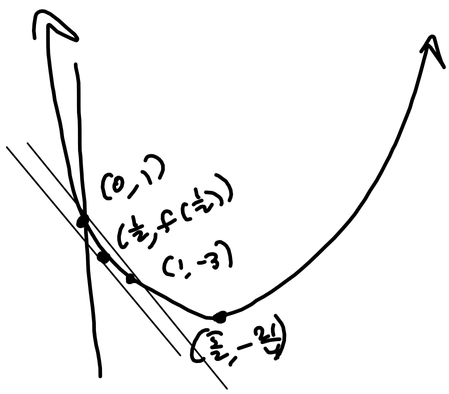
f is polynomial $\Rightarrow$ cnts on $[0,1]$ & difbl on $(0,1)$ (Everywhere!)

$\therefore \exists\, c \in (0,1) \ni f'(x) = m_{avg} = \dfrac{f(1)-f(0)}{1-0} = -3-1 = -4$

$\Rightarrow f'(x) = 2x-5 \overset{\text{set}}{=} -4 \rightarrow$

$2x = 1$

$x = \frac{1}{2}$



x^2-5x+1

$= x^2-5x+\left(\frac{5}{2}\right)^2 - \frac{25}{4} + 1$

$= \left(x-\frac{5}{2}\right)^2 - \frac{21}{4}$