Recall:

Given $f(x)$, we compute $\frac{d}{dx}\left[f^{-1}(x)\right]$ as follows

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \left(f^{-1}\right)'(x) = \frac{1}{f'(f^{-1}(x))}$$

Think about $\arcsin(x) = \sin^{-1}(x)$

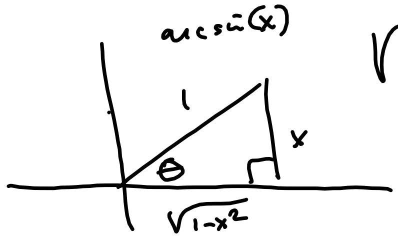
$f(x) = \sin(x)$

$f^{-1}(x) = \arcsin(x)$

$\frac{d}{dx}\left[\sin(x)\right] = \cos(x)$, so

$$\frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\cos(\arcsin(x))} \quad !$$

$$= \frac{1}{\cos\theta}$$



$$\sqrt{1^2 - x^2} = \sqrt{1-x^2}$$

so $\frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\cos(\arcsin(x))} = \frac{1}{\sqrt{1-x^2}}$

New derivative for ya:

$$\frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\sqrt{1-x^2}}$$

Consider

$$\int \arcsin(x)\,dx$$

Let $u = \arcsin(x) \Rightarrow du = \frac{1}{\sqrt{1-x^2}}\,dx$

$dv = dx \Rightarrow v = x$

Integration by Parts:

$$\int u\,dv = uv - \int v\,du =$$

$$= (\arcsin(x))x - \int x\left(\frac{dx}{\sqrt{1-x^2}}\right) = x\arcsin(x) + \frac{1}{2}\int (1-x^2)^{-\frac{1}{2}}(-2x\,dx)$$

$u = 1-x^2 \Rightarrow$
$du = -2x\,dx$

$$\frac{1}{2}\int u^{-\frac{1}{2}}du$$

$$= \frac{1}{2}\cdot 2u^{\frac{1}{2}} + C$$

$$= \boxed{x\arcsin(x) + \sqrt{1-x^2} + C = \int \arcsin(x)\,dx}$$

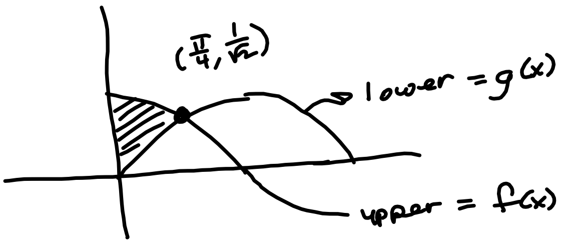
By similar work

$$\boxed{\int \arccos(x)\,dx = x\arccos(x) - \sqrt{1-x^2} + C}$$

Let's do the area bounded by $y = \sin(x)$ & $y = \cos(x)$

& $x = 0$, $x = \frac{\pi}{4}$

Need this (Jesse saw it.)



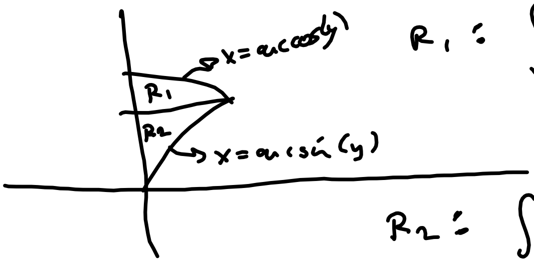
$$\text{Area} = \int (\text{upper} - \text{lower})\,dx = \int_a^b (y_u - y_\ell)\,dx = \int_0^{\frac{\pi}{4}} (\cos(x) - \sin(x))\,dx$$

$$= \Big[\sin(x) + \cos(x)\Big]_0^{\frac{\pi}{4}} = \sin(\tfrac{\pi}{4}) + \cos(\tfrac{\pi}{4}) - (\sin(0) + \cos(0))$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 = \frac{2}{\sqrt{2}} - 1 = \frac{2\sqrt{2}}{\sqrt{2}\sqrt{2}} - 1 = \frac{2\sqrt{2}}{2} - 1$$

$$= \boxed{\sqrt{2} - 1}$$

What about $\int (x_r - x_\ell)\,dy$



R_1: $\int_{\frac{1}{\sqrt{2}}}^{1} \arccos(y)\,dy$

R_2: $\int_0^{\frac{1}{\sqrt{2}}} \arcsin(y)\,dy$

$y = \sin(x) \longrightarrow$

$\arcsin(y) = \arcsin(\sin(x)) = x$ as long as $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$y = \cos(x) \longrightarrow$

$\arcsin(y) = x$ as long as $x \in [0, \pi]$

$$R_1 + R_2 = \int_{\frac{1}{\sqrt{2}}}^{1} \arccos(y)\,dy + \int_0^1 \arcsin(y)\,dy$$

$$\left[y\arccos(y) - \sqrt{1-y^2}\right]_{\frac{1}{\sqrt{2}}}^{1} + \left[y\arcsin(y) + \sqrt{1-y^2}\right]_0^{\frac{1}{\sqrt{2}}}$$

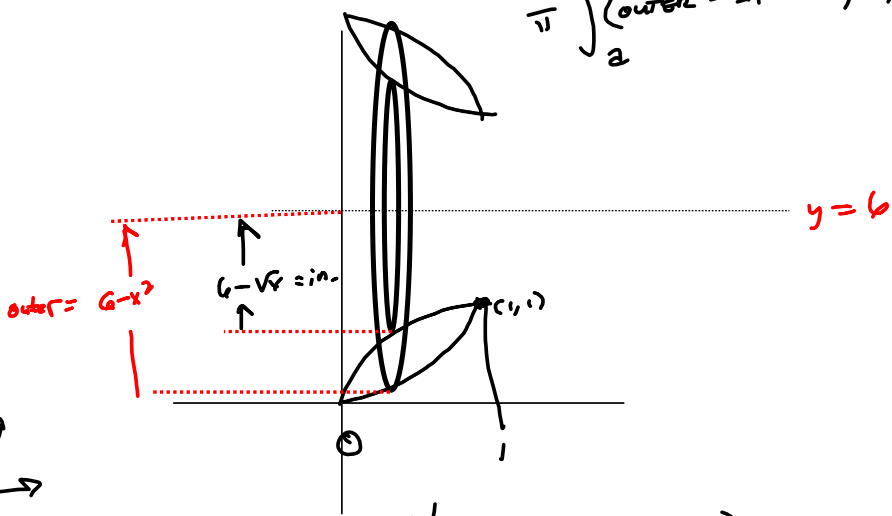
$$= 1\cdot 0 - \sqrt{1-1^2} \boxed{-\left(\frac{1}{\sqrt{2}}\cdot\frac{\pi}{4}\right)} - \sqrt{1-\frac{1}{2}} \boxed{+\frac{1}{\sqrt{2}}\cdot\frac{\pi}{4}} + \sqrt{\frac{1}{2}} - \left(0\cdot\arcsin(0) + \sqrt{1}\right)$$

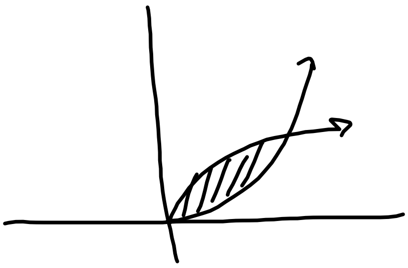
$$= \sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}} + 1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + 1 = \boxed{\sqrt{2} - 1}$$

$y = x^2$, $y = \sqrt{x}$, about $y = 6$

Find volume of the solid of revolution

$$\pi \int_a^b (\text{OUTER}^2 - \text{INNER}^2)\, dx$$





$$\text{Volume} = \pi \int_0^1 \left((6 - x^2)^2 - (6 - \sqrt{x})^2\right) dx$$

$\frac{d}{dx}[b^x] = \ln(b)\, b^x$

$\int b^x dx = \frac{1}{\ln(b)} b^x + C$

$y = 3^x \Rightarrow y' = \ln(3) \cdot 3^x$

$y = 3^{\sin(x)} \Rightarrow y' = \ln(3) \cdot 3^{\sin(x)} \cos(x)$

$\int 3^x dx = \frac{1}{\ln(3)} \cdot 3^x + C$

Tested over

$\int 3^{\sin(x)} dx =$

$\int e^{x^2} dx$

It turns out, there's no closed-form expression!

$u = e^{x^2} \Rightarrow du = 2xe^{x^2} dx$

$dv = dx \Rightarrow v = x$

$\Rightarrow uv - \int v\,du = x e^{x^2} - \int x(2xe^{x^2} dx)$

$u = x \quad du = dx$

$dv = 2xe^{x^2} \Rightarrow v = e^{x^2}$

$xe^{x^2} - xe^{x^2} + \int e^{x^2} du = \int e^{x^2} dx$ No help!

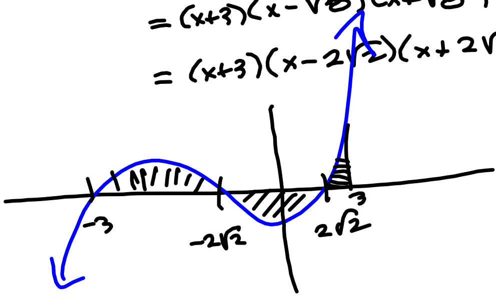
NOT TESTED

#3

Probably will do a polynomial of degree 3 that factors by grouping.

$$x^3+3x^2-8x-24$$
$$=x^2(x+3)-8(x+3)$$
$$=(x+3)(x^2-8)=(x+3)(x^2-\sqrt{8}^{\,2})$$
$$=(x+3)(x-\sqrt{8})(x+\sqrt{8})$$
$$=(x+3)(x-2\sqrt{2})(x+2\sqrt{2})$$

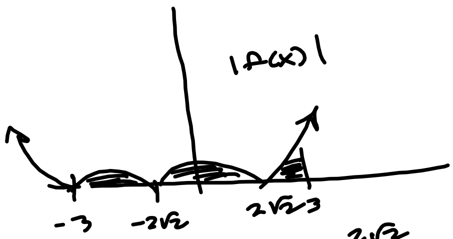
$2(1.42)=2.84$



(signed) $\text{Area}=\int_{-3}^{3} f(x)\,dx$

$$\int_{-3}^{3} f(x)\,dx$$

$$\int_{-3}^{3} |f(x)|\,dx$$



$$\text{Area}=\int_{-3}^{-2\sqrt{2}} f(x)\,dx+\int_{-2\sqrt{2}}^{2\sqrt{2}} -f(x)\,dx+\int_{2\sqrt{2}}^{3} f(x)\,dx$$

$$\int \csc^{18}(x)\cot(x)\,dx$$

$$= -\int \csc^{17}(x)\left(-\csc(x)\cot(x)\,dx\right)$$

$$u = \csc(x) \Longrightarrow du = -\csc(x)\cot(x)\,dx$$

$$= -\int u^{17}\,du = -\frac{u^{18}}{18} + C = \frac{\csc^{18}(x)}{18} + C$$

Logarithmic Differentiation.

Find y'

$$y = (x^2+5x)^{\tan(x)}$$

$$\ln(y) = \ln\left((x^2+5x)^{\tan(x)}\right) = \tan(x)\ln(x^2+5x) \longrightarrow$$

$$\frac{y'}{y} = \sec^2(x)\ln(x^2+5x) + \tan(x)\left(\frac{2x+5}{x^2+5x}\right)$$

$$\Rightarrow y' = \left(\sec^2(x)\ln(x^2+5x) + \tan(x)\left(\frac{2x+5}{x^2+5x}\right)\right)(x^2+5x)^{\tan(x)}$$

$$\frac{d}{dx}[\ln(f(x))] = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$$