**Today:**

**Questions and Section 6.4 Examples**

$$\frac{d}{dx}\left[b^x\right] = \ln(b)\, b^x$$

$$\int b^x\,dx = \frac{1}{\ln(b)} b^x + C$$

My fallback

$$x = e^{\ln(x)}$$

$$b^x = \left(e^{\ln(b)}\right)^x = e^{(\ln(b))x}$$

$$\frac{d}{dx}\left[e^{(\ln b)x}\right] = \ln(b) e^{\ln(b)x} = \ln(b)\, b^x$$

$$\int b^x\,dx = \int e^{\ln(b)x}\,dx = \int e^{\ln(b)x}\frac{du}{\ln(b)} = \frac{1}{\ln(b)}\int e^u\,du$$

$u = \ln(b)x$

$du = \ln(b)\,dx$

$dx = \frac{du}{\ln(b)}$

$$= \frac{1}{\ln(b)} e^u + C$$

$$= \frac{1}{\ln(b)} b^x + C$$

$P(1+i)^{nt}$ = Compound interest $\xrightarrow{\#\text{periods}/yr \to \infty}$ $Pe^{rt}$

$r$ = annual % rate
$m$ = # of compoundings per year
$t$ = " " years

$P$ = Principal = initial amt.

$n = mt$

$i = \frac{r}{m}$

$A = P(1+\frac{r}{m})^{mt} = P(1+i)^n$

$$e = \lim_{x \to 0} (1+x)^{\frac{1}{x}}$$

$$A = P(1+\frac{r}{m})^{mt} = P(1+\frac{1}{\frac{m}{r}})^{\frac{m}{r} \cdot rt} = P\left((1+\frac{1}{\frac{m}{r}})^{\frac{m}{r}}\right)^{rt}$$

As $m \to \infty$, $\frac{1}{\frac{m}{r}} \to 0$

$\lim_{m \to \infty} \frac{r}{m} \to 0$

Let

Let $u = \frac{1}{\frac{m}{r}}$

Then $m \to \infty$

iff $u \to 0$

iff $\frac{1}{\frac{m}{r}} \to 0$

$m \to \infty$ is same as

$\frac{r}{m} \to 0$

$$e = \lim_{x \to 0} (1+x)^{\frac{1}{x}}$$

$$= \lim_{m \to \infty} P\left(1+\frac{r}{m}\right)^{mt} = \lim_{\frac{1}{\left(\frac{r}{m}\right)} \to 0} P\left(\left(1+\frac{1}{\frac{m}{r}}\right)^{\frac{m}{r}}\right)^{rt}$$

$$= \lim_{m \to \infty} P\left(\left(1+\frac{1}{\frac{m}{r}}\right)^{\frac{1}{\frac{1}{\frac{m}{r}}}}\right)^{rt}$$

$$= \lim_{u \to 0} P\left(\left(1+u\right)^{\frac{1}{u}}\right)^{rt}$$

$$= P(e)^{rt} \approx P\left(1+\frac{r}{365}\right)^{365t}$$

$$\left.\begin{array}{l} \lim_{x \to \infty} \left(1+\frac{1}{x}\right)^{x} \\ \lim_{x \to 0} (1+x)^{\frac{1}{x}} \end{array}\right\} \text{Same !}$$

$$\frac{d}{dx}\left[\sin(\underbrace{\ln(x)}_{u})\right] = \cos(\ln(x)) \cdot \frac{1}{x}$$

$$\frac{d}{dx}\left[\frac{1}{\ln(x)}\right] = \frac{d}{dx}\left[(\ln(x))^{-1}\right] = -1\ln(x)^{-2}\left(\frac{1}{x}\right)$$

$$= \frac{-1}{x\ln(x)^2}$$

$$\ln(x)^{-2} \neq \ln(x^{-2})$$

$$\frac{d}{dy}\left[\ln\left(\frac{(2y+1)^5}{\sqrt{y^2+1}}\right)\right] = \frac{d}{dy}\left[\ln\left((2y+1)^5\right) - \ln\left(\sqrt{y^2+1}\right)\right]$$

$$= \frac{d}{dy}\left[5\ln(2y+1) - \frac{1}{2}\ln(y^2+1)\right] = \frac{d}{dy}\left[5\ln(2y+1)\right] - \frac{d}{dy}\left[\frac{1}{2}\ln(y^2+1)\right]$$

$$= 5\frac{d}{dy}\left(\ln(2y+1)\right) - \frac{1}{2}\frac{d}{dy}\left(\ln(y^2+1)\right)$$

$\frac{d}{dy}$ is Linear Operator

$$= \boxed{5\left(\frac{2}{2y+1}\right) - \frac{1}{2}\left(\frac{2y}{y^2+1}\right)}$$

$$(af+bg)' = \frac{d}{dy}\left[af(y)+bg(y)\right]$$

$$= a\frac{d}{dy}\left(f(y)\right) + b\frac{d}{dy}\left[g(y)\right]$$

$$= af' + bg'$$

$(a, b \in \mathbb{R}$ & $f, g$ are difbl$)$

**24.** $y = \log_2(x \log_5 x)$

$$= \frac{1}{\ln(2)}\left[\frac{\frac{d}{dx}(x\log_5(x))}{x\log_5(x)}\right] = \frac{1}{\ln(2)}\left[\frac{\log_5(x)+\frac{1}{\ln(5)}}{x\log_5(x)}\right]$$

$f = x \qquad f' = 1$

$g = \log_5(x) \qquad g' = \frac{1}{\ln 5}\cdot\frac{1}{x}$

$f'g + fg' = \log_5(x) + \cancel{x}\cdot\frac{1}{\ln 5}\cdot\frac{1}{\cancel{x}}$

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Find $y'$ & $y''$

$y = \ln|\sec(x)| \longrightarrow$

$y' = \frac{\sec(x)\tan(x)}{\sec(x)} = \tan(x)$ !

$$\boxed{\int \tan(x)\,dx = \ln|\sec(x)| + C}$$

$\int \frac{\sin(x)}{\cos(x)}\,dx$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$dx = -\frac{du}{\sin(x)}$

$= \int \frac{\sin(x)}{u}\cdot\frac{du}{-\sin(x)}$

$= -\int \frac{du}{u} = -\ln|u| + C$

$= -\ln|\cos(x)| + C$

$= \ln(|\cos(x)|^{-1}) + C$

$= \ln|\cos(x)^{-1}| + C$

$= \ln|\sec(x)| + C$

My way:

$u = \cos(x)$

$du = -\sin(x)\,dx$

$\int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{-\sin(x)\,dx}{\cos(x)}$

$= -\int \frac{du}{u} = -\ln|u| + C$, etc

$\int \sin(2x)\,dx$

$u = 2x$

$du = 2dx$

$= \frac{1}{2}\int \sin(2x)\cdot 2\,dx = \frac{1}{2}(-\cos(2x)) + C$

$$\frac{d}{dx}\left[\ln|x|\right] = \frac{d}{dx}\left[\begin{cases} \ln(x) & \text{if } x>0 \\ \ln(-x) & \text{if } x<0 \end{cases}\right]$$

Stay away from $x=0$.

$$= \begin{cases} \frac{d}{dx}[\ln(x)] & \text{if } x>0 \\ \frac{d}{dx}[\ln(-x)] & \text{if } x<0 \end{cases} = \begin{cases} \frac{1}{x} & \text{if } x>0 \\ \frac{-1}{-x} & \text{if } x<0 \end{cases}$$

$$= \begin{cases} \frac{1}{x} & \text{if } x>0 \\ \frac{1}{x} & \text{if } x<0 \end{cases}$$

i.e. $\int \frac{1}{x}\,dx = \ln|x| + C$

$$\int \frac{\sin(x)}{\cos(x)}\,dx = \int \frac{1}{u}\,du = \ln|u| + C \qquad = -\int \frac{-\sin(x)\,dx}{\cos(x)}$$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$\ldots = \ln|\sec(x)| + C$

Domain & Range

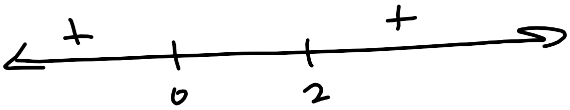
$\mathcal{D}(e^x) = (-\infty, \infty) = \mathcal{R}(\ln(x))$

$\mathcal{R}(e^x) = (0, \infty) = \mathcal{D}(\ln(x))$

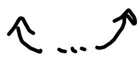
$\mathcal{D}(\ln(x^2 - 2x))$ :

Need $x^2 - 2x > 0$

$x(x-2) > 0$

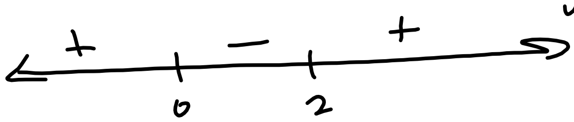



$x^2$



end behavior

want $>0$. "+"




$x$ $(x-2)$

$$\boxed{\mathcal{D} = (-\infty, 0) \cup (2, \infty)}$$

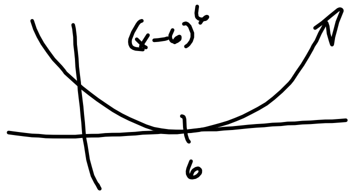
$\mathcal{D}\left(\ln\left(\frac{(x+2)^3(x-7)^2}{(x+5)^5(x-6)^6}\right)\right)$ $\qquad \frac{x^5}{x^{11}} \xrightarrow{x \to \infty} +$

$+ \quad - \quad + \quad + \quad +$

$-5 \quad -2 \quad 6 \quad 7$

$\cancel{\star} \quad =0 \quad \cancel{\star} \quad =0$

$\mathcal{D} = (-\infty, -5) \cup (-2, 6) \cup (6, 7) \cup (7, \infty)$




$\mathcal{D}\left(\sqrt{\frac{(x+2)^3(x-7)^2}{(x+5)^5(x-6)^6}}\right)$

$+$ Y N $\quad$ N $-$ $\quad$ Y $\quad$ Y $+$ $\quad$ N $\quad$ Y $+$ $\quad$ Y $\quad$ Y $+$

$-5 \quad -2 \quad 6 \quad 7$

$\cancel{\star} \quad =0 \quad \cancel{\star} \quad =0$

want $\geq 0$

AND $x \neq -5$ OR $+6$

$\boxed{\mathcal{D} = (-\infty, -5) \cup [2, 6) \cup (6, \infty)}$