S 6.3

3 Theorem If $b > 1$, the function $f(x) = \log_b x$ is a one-to-one, continuous, increasing function with domain $(0, \infty)$ and range $\mathbb{R}$. If $x, y > 0$ and r is any real number, then

1. $\log_b(xy) = \log_b x + \log_b y$
2. $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
3. $\log_b(x^r) = r \log_b x$

§6.4 #17

$x^y = y^x$ Find y'

$\frac{d}{dx}\left[y\ln(x) = x\ln(y) \right]$

$y'\ln(x) + y\left(\frac{1}{x}\right) = 1\ln(y) + x\left(\frac{y'}{y}\right)$

$\frac{d}{dx}[\ln(y)] = \frac{d\ln(y)}{dy} \cdot \frac{dy}{dx} = \frac{1}{y} \cdot y' = \frac{y'}{y}$

$y'\ln(x) - \frac{x}{y}y' = \ln(y) - \frac{y}{x}$ $\longrightarrow$

$y'\left(\ln(x) - \frac{x}{y}\right) = $ SAME $\longrightarrow$

$\boxed{y' = \frac{\ln(y) - \frac{y}{x}}{\ln(x) - \frac{x}{y}}} \cdot \frac{xy}{xy} = \frac{xy\ln(y) - y^2}{xy\ln(x) - x^2}$

WebAssign is Fine with this

Just another way of expressing it.

ADD THIS BONUS QUESTION TO WRITING PROJECT #3!!

Writing Project #3 question 2 stops before asking you to evaluate the integral.

2. i. (5 pts bonus) Evaluate the integral from #2h.

RON POPEIL

Some 6.3 stuff

$2\,|\,32$
$2\,|\,16$
$2\,|\,8$
$2\,|\,4$
2

3–8 Find the exact value of each expression.

3. (a) $\log_2 32$ $= \log_2(2^5) = 5$ (b) $\log_8 2$

4. (a) $\log_5 \frac{1}{125}$ (b) $\ln(1/e^2)$

5. (a) $e^{\ln 4.5}$ (b) $\log_{10} 0.0001$

6. (a) $\log_{1.5} 2.25$ (b) $\log_5 4 - \log_5 500$

7. (a) $\log_{10} 40 + \log_{10} 2.5$
(b) $\log_8 60 - \log_8 3 - \log_8 5$ $= \log_8\left(\frac{60}{15}\right) = \log_8 4$

8. (a) $e^{-\ln 2}$ (b) $e^{\ln(\ln e^3)}$

$\log_8(2) = \log_8(8^{\frac{1}{3}}) = \frac{1}{3}$

$8 = \sqrt[3]{2} = 2^{\frac{1}{3}}$

$\log_{10} 40 + \log 2.5 = \log(40 \cdot 2.5) = \log(100) = \log(10^2) = 2$

$\log_8(4)$ means $4 = 8^x$

$\ln(4) = \ln(8^x) = \ln(8)\,x$

~~$x = \ln(4) - \ln(8)$~~ NO
~~$= \ln(\frac{1}{2}) = -\ln(2)$~~

Instead

$\frac{\ln(4)}{\ln(8)} = x$

$= \frac{\ln(2^2)}{\ln(2^3)} = \frac{2\ln(2)}{3\ln(2)} = \boxed{\frac{2}{3} = x}$

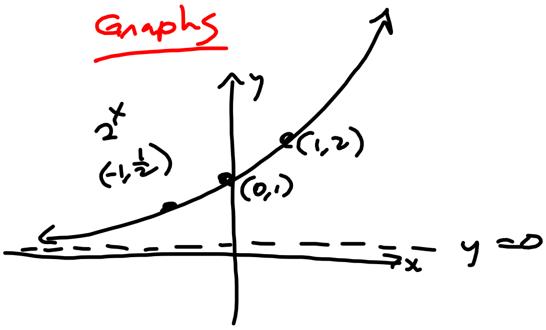
$4 = 8^x$

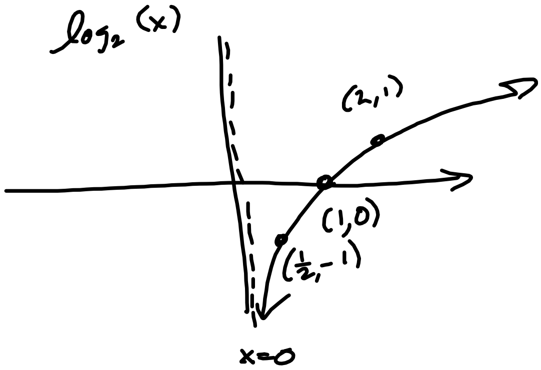
$2^2 = (2^3)^x = 2^{3x} \longrightarrow$

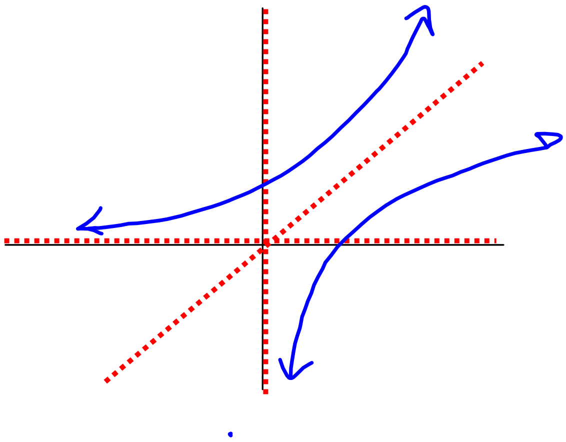
$2 = 3x \longrightarrow$

$\boxed{\frac{2}{3} = x}$

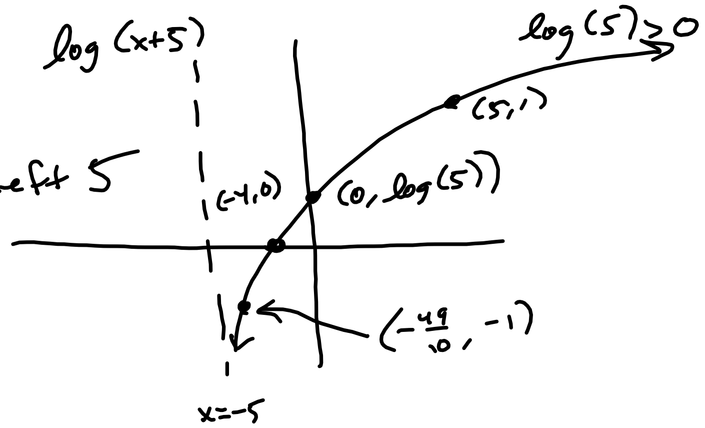
Graphs

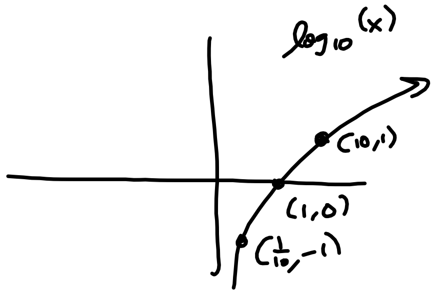






Left 5





$$\frac{1}{10} - 5 = \frac{1-50}{10} = -\frac{49}{10}$$

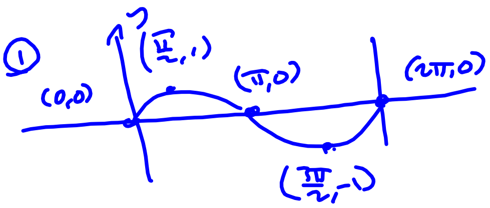
Graphing by transformations

$f(x) = 3\sin\left(\frac{\pi}{6}x - \frac{7\pi}{6}\right) + 11$

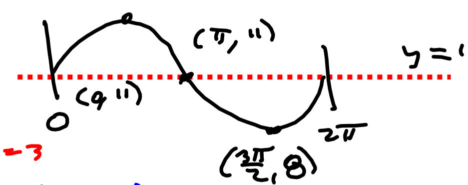
$= 3\sin\left(\frac{\pi}{6}(x-7)\right) + 11$

$y = \sin(x) = f(x)$

1



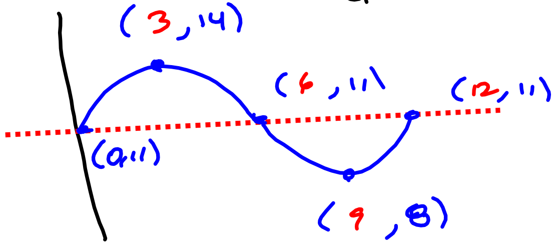
2 $3\sin(x) + 11$



$\frac{6}{\pi} \cdot \frac{\pi}{2} = 3$

3 $3\sin\left(\frac{\pi}{6}x\right) + 11$

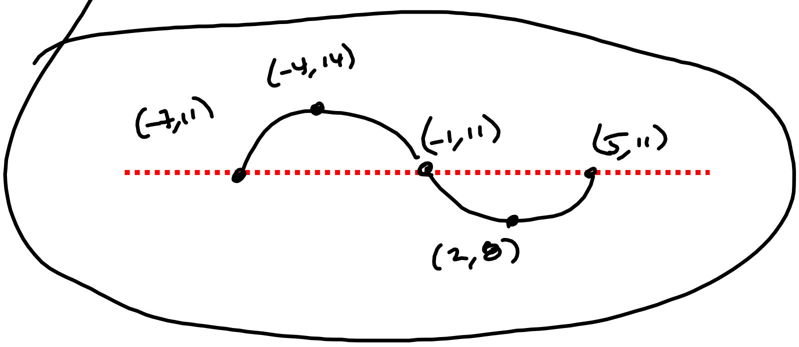
$x \longmapsto \frac{6}{\pi}x$



$\frac{6}{\pi} \cdot \frac{3\pi}{2} = 9$

4 $\boxed{3\sin\left(\frac{\pi}{6}(x-7)\right) + 11}$

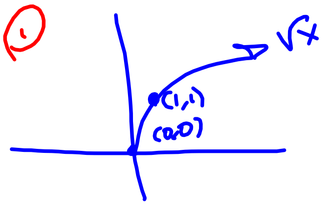
$x \longmapsto x + 7$

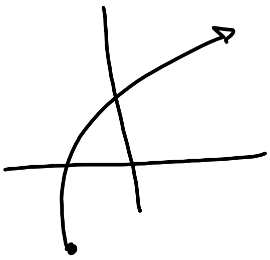


$f(x)=\sqrt{x}$ Graph $g(x) = 7\sqrt{5x+20} - 5$

$= 7\sqrt{5(x+4)} - 5$

1



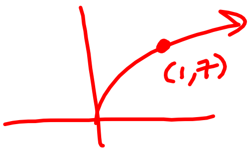


$7\sqrt{5(4)} - 5$

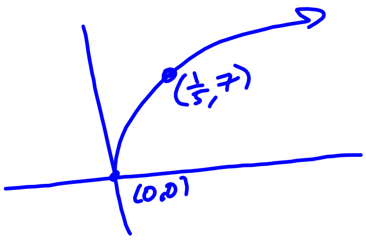
$= 7\sqrt{20} - 5$

$= 14\sqrt{5} - 5$

2 $7\sqrt{x}$ $y \longmapsto 7y$



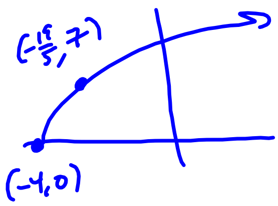
3 $7\sqrt{5x}$ $x \longmapsto \frac{1}{5}x$



(4) $7\sqrt{5(x+4)}$

$x \longmapsto x-4$

$\frac{1}{5} - \frac{4\cdot 5}{5} = \frac{-19}{5}$

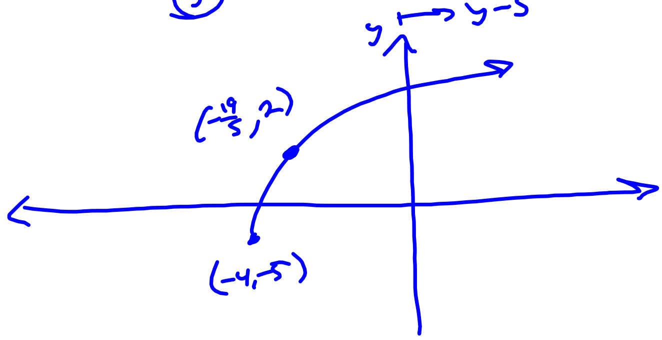


$af(bx+c)+d$

$af\left(b\left(x+\frac{c}{b}\right)\right)+d$

(5) $7\sqrt{5(x+4)} - 5 = g(x)$

$y \longmapsto y-5$



27–36 Solve each equation for x.

27. (a) $e^{7-4x} = 6$ (b) $\ln(3x - 10) = 2$

28. (a) $\ln(x^2 - 1) = 3$ (b) $e^{2x} - 3e^x + 2 = 0$

29. (a) $2^{x-5} = 3$ (b) $\ln x + \ln(x - 1) = 1$

30. (a) $e^{3x+1} = k$ (b) $\log_2(mx) = c$

31. $e - e^{-2x} = 1$

32. $10(1 + e^{-x})^{-1} = 3$

33. $\ln(\ln x) = 1$

34. $e^{e^x} = 10$

35. $e^{2x} - e^x - 6 = 0$

36. $\ln(2x + 1) = 2 - \ln x$

$e^{7-4x} = 6 \Rightarrow$

$\ln(e^{7-4x}) = \ln(6) \Rightarrow$

$7 - 4x = \ln(6)$

$-4x = \ln(6) - 7$

$x = \dfrac{\ln(6) - 7}{-4} = \dfrac{7 - \ln(6)}{4}$

$e^{2x} = (e^{x}) \quad = u^{-}$

$e^{2x} - e^{x} - 6 = 0 \longrightarrow u = e^{x} \longrightarrow$

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0 \longrightarrow$

$u = e^{x} = 3$ or $u = e^{x} = \cancel{-2}$ Nope

$\ln(e^{x}) = \ln(3)$

$\boxed{x = \ln(3)}$

Nope

$e^{x} - e^{-2x} = 1$

$e^{x} - \frac{1}{e^{2x}} = 1$

$\frac{e^{3x} - 1}{e^{2x}} = 1$

$e^{3x} - 1 = e^{2x}$

$e^{3x} - e^{2x} - 1 = 0$

$e^{3x} - e^{2x} - e^{0} = 0$

I don't see it.

$\ln(e^{x} - e^{-2x}) = 0$

$e^{x}[1 - e^{-3x}] = 1$

$1 - e^{-3x} = e^{-x}$

$e^{-2x}[e^{x} - e^{-2x} = 1]$

$e^{-x} - e^{-4x} = e^{-2x}$

$e - e^{-2x} = 1$

$-e^{-2x} = 1 - e$

$e^{-2x} = e - 1$

$-2x = \ln(e - 1)$

$x = -\frac{\ln(e-1)}{2}$