Differentiate the function.

$$H(z) = \ln\left(\sqrt{\frac{c^2 - z^2}{c^2 + z^2}}\right)$$

Recall: $\frac{d}{dx}\left[\ln(f(x))\right] = \frac{f'(x)}{f(x)}$

$\ln(ab) = \ln(a) + \ln(b)$

$\ln(a^b) = b\ln(a)$

$$\boxed{\frac{2c^2 z}{z^4 - c^4}}$$

$H(z) = \frac{1}{2}\left[\ln(c^2 - z^2) - \ln(c^2 + z^2)\right]$

$\Rightarrow H'(z) = \frac{1}{2}\left[\frac{-2z}{c^2 - z^2} - \frac{2z}{c^2 + z^2}\right]$ was accepted

$= \frac{1}{2}\left[-2z(\right.$

on a test, I'd probably want the 2's canceled.

$\Rightarrow H'(z) = \frac{-z}{c^2 - z^2} - \frac{z}{c^2 + z^2}$

Jesse's answer

$$\frac{d}{dx}\left[\ln|x|\right]$$

$$= \frac{d}{dx}\left[\begin{cases} \ln(x) & \text{if } x>0 \\ \ln(-x) & \text{if } x<0 \end{cases}\right]$$

$$= \begin{cases} \frac{1}{x} & \text{if } x>0 \\ \frac{1}{-x}(-1) & \text{if } x<0 \end{cases} = \frac{1}{x} \quad (x \neq 0)$$

$$\Rightarrow \int \frac{1}{x}\,dx = \ln|x| + C \quad (x \neq 0)$$

Note: $\int_{-1}^{1} \frac{1}{x}\,dx \quad \nexists$

Last Bit of NEW!

Logarithmic Differentiation

We handle: $\frac{d}{dx}\left[f(x)^{g(x)}\right]$

Find $\frac{d}{dx}\left[\cos(x)^{x^2-2x}\right]$

$\ln(xy) = \ln(x) + \ln(y)$

$\ln(a^b) = b\ln(a)$

$y = \cos(x)^{x^2-2x} \longrightarrow$

$\frac{d}{dx}\left[\ln(y) = \ln\left(\cos(x)^{x^2-2x}\right) = (x^2-2x)\ln(\cos(x))\right] \longrightarrow$

$\frac{y'}{y} = (2x-2)\ln(\cos(x)) + (x^2-2x)\left(\frac{-\sin(x)}{\cos(x)}\right) \longrightarrow$

$$\boxed{y' = \left[(2x-2)\ln(\cos(x)) - (x^2-2x)\tan(x)\right]\cos(x)^{x^2-2x}}$$

$\frac{d}{dx}\left[\frac{x^2-4x+3}{x+1}\right] = \frac{d}{dx}\left[\frac{(x-1)(x-3)}{x+1}\right]$

$y = \frac{(x-1)(x-3)}{x+1} \longrightarrow$

$\ln(y) = \ln(x-1) + \ln(x-3) - \ln(x+1)$

$$\rightarrow \frac{y'}{y} = \frac{1}{x-1} + \frac{1}{x-3} - \frac{1}{x+1} \rightarrow$$

$$y' = \left(\frac{1}{x-1} + \frac{1}{x-3} - \frac{1}{x+1}\right)\left(\frac{(x-1)(x-3)}{x+1}\right)$$

$$= \left(\frac{(x-3)(x+1) + (x-1)(x+1) - (x-1)(x-3)}{(x-1)(x-3)(x+1)}\right)\left(\frac{x^2-4x+3}{x+1}\right)$$

$$= \left(\frac{x^2-2x-3 + x^2-1-(x^2-4x+3)}{(x-1)(x-3)(x+1)}\right)\left(\sim\right)$$

$$= \left(\frac{x^2+2x-7}{\cancel{(x-1)}\cancel{(x-3)}(x+1)}\right)\left(\frac{\cancel{(x-1)(x-3)}}{x+1}\right)$$

$$= \frac{x^2+2x-7}{(x+1)^2} = y' .$$ Alternate: $\frac{d}{dx}\left[\frac{x^2-4x+3}{x+1}\right] =$

$$= \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} = \frac{(2x-4)(x+1) - (x^2-4x+3)(1)}{(x+1)^2}$$

$$= \frac{2x^2 - 2x - 4 - x^2 + 4x - 3}{(x+1)^2} = \frac{x^2+2x-7}{(x+1)^2}$$

Also good for

$$\frac{d}{dx}\left[\frac{(x-1)^3(4x+2)^7}{\sqrt[5]{x^2-13}}\right]$$

$$\ln(y) = 3\ln(x-1) + 7\ln(4x+2) - \frac{1}{5}\ln(x^2-13)$$

$$\Rightarrow y' = \left(3\left(\frac{1}{x-1}\right) + 7\left(\frac{4}{4x+2}\right) - \frac{1}{5}\left(\frac{2x}{x^2-13}\right)\right)\frac{(x-1)^3(4x+2)^7}{\sqrt[5]{x^2-13}}$$

On time-control test: this is

~~perfeck~~

~~poifick~~

perfect

Change of Base

$$y = \log_b(x)$$

$$b^y = b^{\log_b(x)} = x$$

$$\log_a(b^y) = \log_a(x)$$

$$y \log_a(b) = \log_a(x)$$

$$y = \frac{\log_a(x)}{\log_a(b)} = \log_b(x)$$

Biggest Application!

CONVERT TO $\ln(x)$

$$\frac{d}{dx}\left[\log_7(x)\right] = \frac{d}{dx}\left[\frac{\ln(x)}{\ln(7)}\right] = \frac{1}{\ln(7)} \cdot \frac{1}{x} = \frac{1}{x\ln(7)}$$

$$= \frac{1}{(\ln(7))x}$$

Consider $f(x) = 3 \cdot e^x$

$\Longrightarrow f(0) = 3 \cdot e^0 = 3,$

So for the model $N_0 e^{kx}$, where

$N_0 =$ initial population

& k is the relative growth rate.

A population is growing at 5% relative growth rate. The initial population is 10,000. Then

$$Pop = N_0 e^{kx} = 10000 e^{.05x}$$

The $\frac{1}{2}$-life of Carbon-14 is 5700 yrs.
17% of natural C-14 is present in a firepit.
C-12 is non-radioactive.

The $\frac{1}{2}$-life of C-14 (C_{14}) is 5700 yrs.

$N(t) = N_0 e^{kt}$

$N(5700) = N_0 e^{5700k} = \frac{1}{2} N_0$ Solve for k:

$\longrightarrow e^{5700k} = \frac{1}{2}$

$\ln(e^{5700k}) = 5700k = \ln(\frac{1}{2}) = \ln(2^{-1}) = -\ln(2)$

$\longrightarrow k = \frac{-\ln(2)}{5700}$.

Now, how old is a sample w/ 17% of C_{14} remaining?

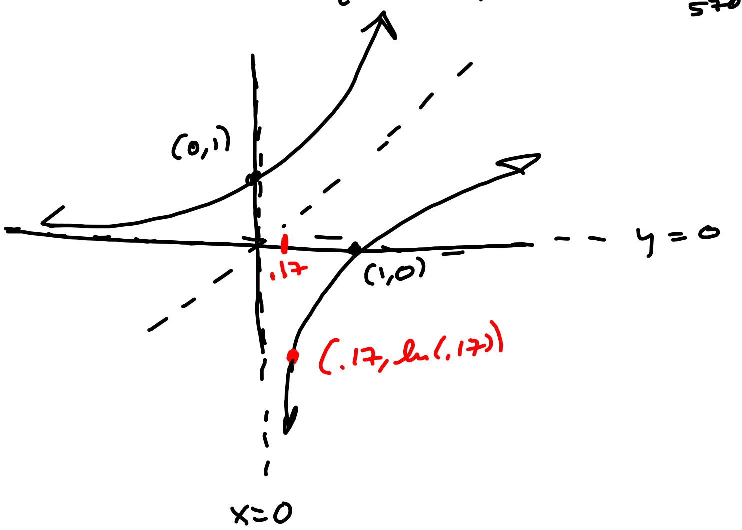
$N_0 e^{kt} = .17 N_0$

$e^{kt} = .17$ SOLVE FOR t:

$kt = \ln(.17)$

$t = \frac{\ln(.17)}{k} = \frac{\ln(.17)}{\frac{-\ln(2)}{5700}} = \frac{5700 \ln(.17)}{-\ln(2)}$

$\approx$ 14571.44209 yrs



100%	0 yrs
50%	5700 yrs
25%	11400 yrs
12.5%	17100 yrs