$$y = \frac{x^2-4x+3}{x+1} = \frac{(x-3)(x-1)}{x+1} \Longrightarrow$$

$$\ln(y) = \ln(x-3) + \ln(x-1) - \ln(x+1) \Longrightarrow$$

$$\frac{y'}{y} = \frac{1}{x-3} + \frac{1}{x-1} - \frac{1}{x+1} \Longrightarrow$$

$$y' = \left(\frac{1}{x-3} + \frac{1}{x-1} - \frac{1}{x+1}\right)\left(\frac{x^2-4x+3}{x+1}\right)$$

$$= \left(\frac{(x-1)(x+1) + (x-3)(x+1) - (x-1)(x-3)}{(x-3)(x-1)(x+1)}\right)\left(\frac{x^2-4x+3}{x+1}\right)$$

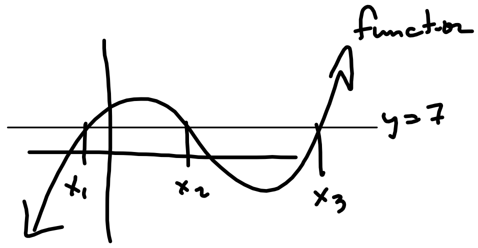
$$= \left(\frac{x^2-1+x^2-2x-3-x^2+4x-3}{LCD}\right)(y)$$

$$= \left(\frac{x^2+2x-7}{(x-3)(x-1)(x+1)}\right)\left(\frac{(x-3)(x-1)}{x+1}\right)$$

$$= \frac{x^2+2x-7}{(x+1)^2}$$

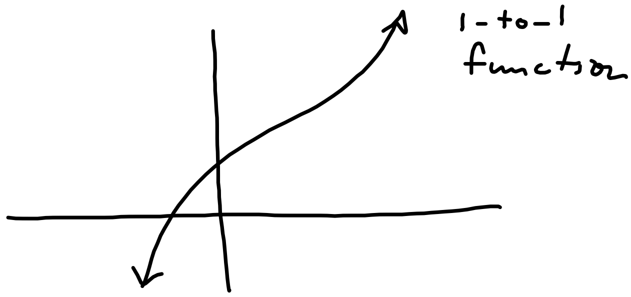
$f : A \to B$ is a <u>function</u> if for each $x \in A =$ Domain of f, $\exists$ exactly one $y \in B =$ Range of f. VERTICAL LINE TEST

1-to-1 $f : A \to B$ is 1-to-1 if $\forall\, y \in \mathcal{R}$ $\exists$ exactly one $x \in \mathcal{D}$. HORIZONTAL LINE TEST



$f(x_1) = f(x_2) = f(x_3) = 7$

$\longrightarrow$ <u>NOT</u> 1-to-1



1-to-1 function has an inverse relation that is also a function. The horizontal line test for *f* is the vertical line test for the inverse f^{-1}

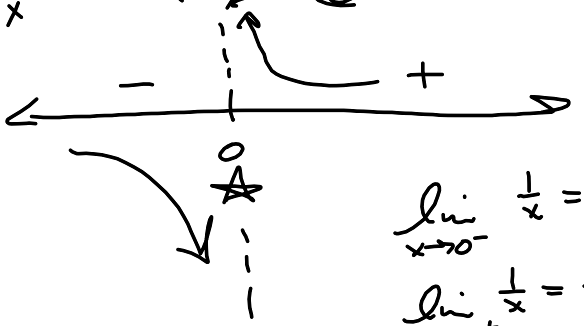
The graph of the inverse is obtained by reflecting the graph of *f* about the line *y* = *x*.

If *f* is cnts, then so is its inverse.

If *f* is difbl, then so is its inverse (assuming $f'(x)$ **.)**

if *f* is increasing (decreasing), so is its inverse.

$\frac{1}{x} \neq 0$ & ☆ @ $x=0$



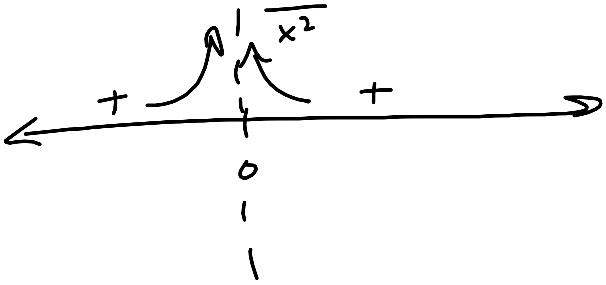
$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

$\lim_{x \to 0^+} \frac{1}{x} = +\infty$

$\frac{1}{BIG}$ is small

$\lim_{x \to \infty} \frac{1}{x} = 0$

$\frac{1}{x^2}$



$\lim_{x\to\infty} \frac{1}{x} = 0$

Pf

Let $\varepsilon > 0$ be given

want $\frac{1}{x} < \varepsilon \Rightarrow$

$\frac{1}{\varepsilon} < x$

Let $M \geq \frac{1}{\varepsilon}$. Then let $x > M \geq \frac{1}{\varepsilon}$

Then $\frac{1}{x} < \frac{1}{\frac{1}{\varepsilon}} = \varepsilon$

$\boxed{D}$ $\lim_{x\to\infty} f(x) = 0$ means $\exists\ M > 0 \ni$ if $x > M$, then $|f(x)| < \varepsilon$ for any $\varepsilon > 0$.

$\boxed{D}$ $\lim_{x\to c} f(x) = \infty$ means that $\forall\ M > 0$, $\exists\ \delta > 0 \ni$ $|x-c| < \delta \Rightarrow f(x) > M$.

Challenge:

Response:

$\lim_{x\to 0^+} \frac{1}{x} = +\infty$

Pf Let $M > 0$ be given.

Scratch:

want $\delta \ni |x-0| < \delta \Rightarrow$

$\frac{1}{x} > M$

$\frac{1}{M} > x > 0$

Define $\delta = \frac{1}{M}$. Then $0 < |x-0| < \delta \Rightarrow$

$x < \frac{1}{M} \Rightarrow M < \frac{1}{x}$ ∎

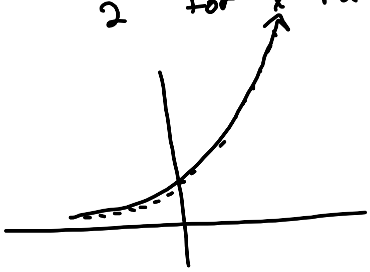
$\lim \frac{1}{x^2} = 0$. Let $M > 0$ be given

Want $\frac{1}{x^2} > M \Rightarrow \frac{1}{M} > x^2$. ~~Define $\delta = \frac{1}{M}$~~

$\Rightarrow \frac{1}{\sqrt{M}} > x$. Define $\delta = \frac{1}{\sqrt{M}}$

S 6.2 - Exponential Functions and the Natural Exponential Function.

2^x for x rational



b^x, $b > 1$ is cnts & difbl

No holes

smooth.

$2^{1.72} < 2^{\sqrt{3}} < 2^{1.74}$

$2^{1.73205080} < 2^{\sqrt{3}} < 2^{1.73205081}$

1.7320508075688772935274463415058723669428052538103806280558

$b > 0$

$0 < b < 1$ Decay $\quad b = 1$ $y = b$ $\quad b > 1$ Growth

$b^{x+h} = b^x b^h$

$$\frac{d}{dx}\left[b^x\right] = \lim_{h\to 0} \frac{b^{x+h} - b^x}{h} = \lim_{h\to 0} \frac{b^x\left[b^h - 1\right]}{h} = b^x \lim_{h\to} \frac{b^h - 1}{h}$$

$$= b^x \cdot \ln(b) \; !$$

is proportional to b^x !

TRY $b = 2$: $\frac{2^h - 1}{h} \xrightarrow{h \to 0}$ 0.6931471806 $\approx \ln(2)$

$\frac{3^h - 1}{h} \xrightarrow{h \to 0}$ 1.098612289 $\approx \ln(3)$

This suggests there is a number between 2 and 3 such that the limit

$$\lim_{h\to 0} \frac{b^h - 1}{h} = 1 .$$

$\boxed{D}$ e is the number such that $\lim_{h\to 0} \frac{b^h - 1}{h} = 1$

Then $\frac{d}{dx}\left[e^x\right] = e^x$!

The natural base 'e' for "Euler"

$$\frac{d}{dx}\left[e^{\sin(x)}\right] = e^{\sin(x)} \cdot \cos(x) \text{ by chain rule.}$$

$$\frac{df}{dg} \cdot \frac{dg}{dx}$$

$$\left.\begin{array}{l} f(x) = e^x \\ g(x) = \sin(x) \end{array}\right\} e^{\sin(x)} = f(g(x))$$

$\boxed{D}$ $\ln(x)$ is the natural logarithm of x

$\boxed{D}$ $\log_b(x)$ is the logarithm to the base b of x

$(\ln(x) = \log_e(x))$

$\log_b(x) = y$ means $x = b^y$

$\log_2(8) = \log_2(2^3) = 3$

$\log_7(49) = 2$

$(a^b)^c = a^{bc}$, $a^b a^c = a^{b+c}$, $\frac{a^b}{a^c} = a^{b-c}$

$\log_b(xy) = \log(x) + \log(y)$

$\log_b(x^y) = y\log_b(x)$

$\log_7\left(\frac{x^2y^3}{\sqrt{z}}\right) = 2\log_7(x) + 3\log_7(y) - \frac{1}{2}\log_7(z)$

$\frac{d}{dx}\left[3^x\right] = 3^x \lim_{h\to 0}\left(\frac{3^h-1}{h}\right) = 3^x \ln(3)$ by Maple.

$\log_b(x)$ & b^x are inverses!

$\log_b(b^x) = x \qquad b^{\log_b(x)} = x$

and

$3 = e^{\ln(3)}$. Right?

$\frac{d}{dx}\left[3^x\right] = \frac{d}{dx}\left[\left(e^{\ln(3)}\right)^x\right] = \frac{d}{dx}\left[e^{\ln(3)\cdot x}\right] = e^{\ln(3)\cdot x} \cdot \ln(3)$!

$= \left(e^{\ln(3)}\right)^x \cdot \ln(3) = 3^x \cdot \ln(3) = \ln(3)\cdot 3^x$

$$\boxed{\begin{array}{ll} \frac{d}{dx}\left[b^x\right] = \ln(b)\cdot b^x \quad \& & \int b^x\,dx = \frac{1}{\ln(b)}b^x + C \\ \frac{d}{dx}\left[e^x\right] = e^x & \int e^x\,dx = e^x + C \end{array}}$$

$y = f(x) = e^x \implies$

$\implies f^{-1}(x) = \ln(x)$

We want the derivative of $\ln(x)$

$y = \ln(x) \longrightarrow$

$e^y = e^{\ln(x)} = x$

$\frac{d}{dx}\left[e^y\right] = e^y \frac{dy}{dx} = 1 \implies$

$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{e^{\ln(x)}} = \frac{1}{x}$

i.e., $\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x}$!

Chain Rule:

$\frac{d}{dx}\left[\ln(f(x))\right] = \frac{1}{f(x)} \cdot f'(x) = \frac{f'(x)}{f(x)}$!

$\int \frac{f'(x)}{f(x)}\,dx = \ln(x) + C$!